Neutrino mixing

Can $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$?

If this happens:

- neutrinos have mass
- physics beyond the (perturbative) Standard Model participates

Outline:

- description/review of mixing phenomenology
- possible experimental signatures
- short review of existing experimental results
Analogy with $K^0 \leftrightarrow \bar{K}^0$ mixing

We produce flavor eigenstates...

\[
\begin{align*}
\text{strong interaction produces a } K^0 \\
\text{weak interaction produces a } \nu_\mu
\end{align*}
\]

…but the mass eigenstates are what propagate “sensibly” without mixing:

\[
\begin{align*}
|K_S(\tau)\rangle &= |K_S(0)\rangle e^{-\Gamma_S \tau} e^{-im_s c^2 \tau / \hbar} \\
|K_L(\tau)\rangle &= |K_L(0)\rangle e^{-\Gamma_L \tau} e^{-im_L c^2 \tau / \hbar} \\
|v_1(\tau)\rangle &= |v_1(0)\rangle e^{-im_1 c^2 \tau / \hbar} \\
|v_2(\tau)\rangle &= |v_2(0)\rangle e^{-im_2 c^2 \tau / \hbar} \\
|v_3(\tau)\rangle &= |v_3(0)\rangle e^{-im_3 c^2 \tau / \hbar}
\end{align*}
\]
Analogy with $K^0 \leftrightarrow \bar{K}^0$ mixing

We know $K_L \neq K^0$, etc. Perhaps $\nu_1 \neq \nu_e$ or $\nu_2 \neq \nu_\mu$ or $\nu_3 \neq \nu_\tau$?

Rewrite the production eigenstates as linear combinations of mass eigenstates:

$$
\begin{pmatrix}
K^0 \\
\bar{K}^0
\end{pmatrix} = U_{2\times2} \begin{pmatrix}
K_S \\
K_L
\end{pmatrix} \quad \begin{pmatrix}
K_S \\
K_L
\end{pmatrix} = (U_{2\times2}^T)^* \begin{pmatrix}
K^0 \\
\bar{K}^0
\end{pmatrix}
$$

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix} \quad \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix} = (U^T)^* \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
$$

$U$ is the Maki-Nakagawa-Sakata matrix.
Maki-Nakagawa-Sakata mixing matrix

Parameterize mixing with three angles and one phase:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{+i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[\begin{align*}
\nu_\mu & \leftrightarrow \nu_\tau \\
\nu_e & \leftrightarrow \nu_\tau \\
\nu_e & \leftrightarrow \nu_\mu
\end{align*}\]

\[c_{12} \equiv \cos\theta_{12}, \quad s_{12} \equiv \sin\theta_{12} \ldots \quad \delta \neq 0: \text{CP violation}\]

This form is convenient if only two neutrino species mix. (CP violation requires that all three mix.)
Analogy with $K^0 \leftrightarrow \bar{K}^0$ mixing

\[ K^0 \leftrightarrow \bar{K}^0 \]

\[ \nu_\mu \leftrightarrow \nu_e \]

\[ \begin{array}{c}
K^0 \\
ds \\
w \\
d \\
K^0 \\
u, c, t \\
u, c, t \\
w \\
K^0 \\
\end{array} \]

\[ |K(\tau = 0)\rangle = |K^0\rangle \sim |K_S\rangle + |K_L\rangle \] (maximal mixing!)

\[ |K(\tau)\rangle = |K_S\rangle e^{-\Gamma_s \tau} e^{-im_{S}c^2\tau/\hbar} + |K_L\rangle e^{-\Gamma_{L} \tau} e^{-im_{L}c^2\tau/\hbar} \]

\[ \sim |K_S\rangle e^{-\Gamma_s \tau} + |K_L\rangle e^{-\Gamma_{L} \tau} e^{-i\Delta m c^2\tau/\hbar} \] ($\Delta m = m_L - m_S$)

$K_L$ phase rotates (relative to $K_S$ phase) $\sim 30^\circ$ per $10^{-10}$ sec.

Recall: $|K^0\rangle \sim |K_S\rangle + |K_L\rangle$ and $|\bar{K}^0\rangle \sim |K_S\rangle - |K_L\rangle$. 
Analogy with $K^0 \leftrightarrow \bar{K}^0$ mixing

$v_\mu \leftrightarrow v_e$

$v_1, v_3$ phases rotate relative to $v_2$ phase if masses are unequal.
Highly contrived example:

\[ m_1 = 0.1 \text{ eV}, \ m_2 = 0.3 \text{ eV}, \ m_3 = 0.4 \text{ eV} \]
\[ \theta_{12} = 10^\circ, \ \theta_{13} = 20^\circ, \ \theta_{23} = 30^\circ, \ \delta = 0^\circ \]

Radius of circle = |amplitude|; line indicates \( Arg(\text{amplitude}) \)

\[ P = 8.73 \times 10^{-34} \quad P = 1. \quad P = 1.1 \times 10^{-32} \]

\[ \langle \nu \mu | \nu (0) \rangle = 1 \]

\[ n_1, \quad m_1 = 0.1 \text{ eV} \quad q_{\theta 2} = 10. \text{ deg} \]
\[ n_2, \quad m_2 = 0.3 \text{ eV} \quad q_{\theta 3} = 20. \text{ deg} \]
\[ n_3, \quad m_3 = 0.4 \text{ eV} \quad q_{\theta 3} = 30. \text{ deg} \]

\[ \langle \nu_1 | \nu (0) \rangle = -0.32 \quad \langle \nu_2 | \nu (0) \rangle = 0.82 \quad \langle \nu_3 | \nu (0) \rangle = 0.47 \]
\[ \frac{d\phi_1}{d\tau} = \frac{m_1 c^2}{\hbar} = 8.7^\circ/10^{-15} \text{ sec}; \quad \frac{d\phi_2}{d\tau} = 26.1^\circ/10^{-15} \text{ sec}; \quad \frac{d\phi_3}{d\tau} = 34.8^\circ/10^{-15} \text{ sec} \]

\[ t = 1.65 \times 10^{-15} \text{ sec} \]

\[ \langle \nu_\mu | \nu(t) \rangle \neq 1 \]

\[
\begin{array}{ccc}
\nu_e & | P = 0.0341 \rangle & \nu_m & | P = 0.961 \rangle & \nu_t & | P = 0.00445 \rangle \\
\end{array}
\]

\[
\begin{array}{ccc}
\nu_1 & m_1 = 0.1 \text{ eV} & \nu_2 & m_2 = 0.3 \text{ eV} & \nu_3 & m_3 = 0.4 \text{ eV} \\
q_{12} = 10. \text{ deg} & q_{13} = 20. \text{ deg} & q_{23} = 30. \text{ deg} \\
\end{array}
\]

\( \nu_1, \nu_2, \nu_3 \) phases have changed: heavier species phase-rotate more rapidly.
Mathematica animation for

\[ m_1 = 0.1 \text{ eV}, \quad m_2 = 0.3 \text{ eV}, \quad m_3 = 0.4 \text{ eV} \]
\[ \theta_{12} = 10^\circ, \quad \theta_{13} = 20^\circ, \quad \theta_{23} = 30^\circ, \quad \delta = 0^\circ \]

(http://web.hep.uiuc.edu/home/g-gollin/neutrinos/amplitudes1.nb)
ν oscillations in a beam with energy $E_\nu$ ...

$x = \text{distance from production} \quad t = \text{time since production}$

\begin{align*}
x_\mu & \equiv (\vec{x}, ct) \quad \text{rest frame: } (0, c\tau) \\
p_\mu & \equiv (\vec{p}, E_\nu / c) \quad \text{rest frame: } (0, mc) \\
p_\mu x^\mu & = -m_i c^2 \tau \quad \text{all frames (Lorentz scalar)} \\
& = px - E_\nu t \quad \text{lab frame}
\end{align*}

\begin{align*}
p & = \sqrt{(E_\nu / c)^2 - (m_i c)^2} \approx E_\nu / c - m_i^2 c^3 / (2E_\nu) ; \quad x \approx ct
\end{align*}

\begin{align*}
\therefore & \quad e^{-im_i c^2 \tau / \hbar} = e^{i(px - E_\nu t / \hbar)} \approx e^{-im_i^2 c^3 x / (2E_\nu \hbar)}
\end{align*}

\begin{align*}
|\nu_i (\tau)\rangle = |\nu_i (0)\rangle e^{-im_i c^2 \tau / \hbar} & \quad \rightarrow \quad |\nu_i (x)\rangle = |\nu_i (0)\rangle e^{-im_i^2 c^3 x / (2E_\nu \hbar)}
\end{align*}
$\nu_\mu$ oscillations in a beam...

\[ |\nu(x=0)\rangle = |\nu_\mu\rangle = U_{21}^\ast |\nu_1\rangle + U_{22}^\ast |\nu_2\rangle + U_{23}^\ast |\nu_3\rangle \]

\[ |\nu(x)\rangle \sim U_{21}^\ast |\nu_1\rangle + U_{22}^\ast |\nu_2\rangle e^{-i\Delta m_{21}^2 c^3 x/(2E\hbar)} + U_{23}^\ast |\nu_3\rangle e^{-i\Delta m_{31}^2 c^3 x/(2E\hbar)} \]

\[ \Delta m_{21}^2 = m_2^2 - m_1^2 \quad \Delta m_{31}^2 = m_3^2 - m_1^2 \quad \text{(factored out } e^{-im_1^2 c^3 x/(2E\hbar)} \text{).} \]

\[ \frac{d\phi_{21}}{dx} = \frac{\Delta m_{21}^2 c^3}{2E\hbar} \approx \frac{\Delta m_{21}^2}{E} \cdot (145^\circ \text{ per km}) \text{ for } \Delta m_{21}^2 \text{ in (eV)}^2, \ E \text{ in GeV} \]

Relative phases of $\nu_1$, $\nu_2$, $\nu_3$ coefficients change: $\nu_\mu \rightarrow \text{other stuff}$
Mathematica animation for

\[ \Delta m_{21}^2 = 0.3 \text{ eV}^2, \quad \Delta m_{31}^2 = 0.6 \text{ eV}^2 \]
\[ \theta_{12} = 5^\circ, \quad \theta_{13} = 10^\circ, \quad \theta_{23} = 15^\circ, \quad \delta = 0^\circ \]

(http://web.hep.uiuc.edu/home/g-gollin/neutrinos/amplitudes2.nb)
Two-flavor mixing

Results are often analyzed with the simplifying assumption that only two of the three $\nu$ species mix.

For example: $\nu_e \leftrightarrow \nu_\mu$ ...

$$|\nu(x = 0)\rangle = |\nu_\mu\rangle = -\sin \theta_{12} |\nu_1\rangle + \cos \theta_{12} |\nu_2\rangle$$

$$|\nu(x)\rangle = -\sin \theta_{12} |\nu_1\rangle + \cos \theta_{12} |\nu_2\rangle e^{-i\Delta m^2_{12}c^3x/(2Eh)}$$

$$P(\nu_\mu \rightarrow \nu_e ; x) = \left| \langle \nu_e | \nu(x) \rangle \right|^2$$

$$= \left| \langle \cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2 | -\sin \theta_{12} \nu_1 + \cos \theta_{12} e^{-i\Delta m^2_{12}c^3x/(2Eh)} \nu_2 \rangle \right|^2$$

$$= (\cos \theta_{12} \sin \theta_{12})^2 \left| 1 - e^{-i\Delta m^2_{12}c^3x/(2Eh)} \right|^2$$
Two-flavor mixing

\[ P(\nu_\mu \rightarrow \nu_e; x) = (\cos \theta_{12} \sin \theta_{12})^2 \left| 1 - e^{-i\Delta m^2 c^3 x / (2E\hbar)} \right|^2 \]

\[ = \frac{1}{2} \sin^2 (2\theta_{12}) \left[ 1 - \cos \left( \frac{\Delta m^2 c^3 x}{2E\hbar} \right) \right] \]

\[ = \sin^2 (2\theta_{12}) \sin^2 \left( \frac{\Delta m^2 c^3 x}{4E\hbar} \right) \]

\[ \approx \sin^2 (2\theta_{12}) \sin^2 \left( 1.27 \Delta m^2 \frac{x}{E} \right) \]

units: \( \Delta m^2 \) in eV\(^2\), \( x \) in km, \( E \) in GeV.

\[ P(\nu_e \rightarrow \nu_e; x) = 1 - P(\nu_\mu \rightarrow \nu_e; x) \]
Mathematica animation for

\[ \Delta m_{21}^2 = 0.3 \text{ eV}^2 \]

\[ \theta_{12} = 15^\circ, \ \theta_{13} = 0^\circ, \ \theta_{23} = 0^\circ, \ \delta = 0^\circ \]

(http://web.hep.uiuc.edu/home/g-gollin/neutrinos/amplitudes3.nb)
Those confusing plots for two-flavor mixing...

\[ P(\nu_\mu \rightarrow \nu_e; L) \approx \sin^2(2\theta_{12}) \sin^2\left(1.27\Delta m_{21}^2 \frac{L}{E}\right) \]

A search experiment sets a limit (or measures!!) the mixing probability \( P \), but little else, at the present time.

**Green** curve is contour of fixed probability to observe oscillation \( L \) for an event is uncertain due to length of \( \pi \rightarrow \mu \) decay/drift region.

**Large** \( \Delta m^2 \): uncertainty in \( L/E \) corresponds to several oscillations.

\( P \) determined by experiment is an average over several oscillations and is insensitive to \( \Delta m^2 \) in this case...
More on those confusing plots...

\[ P\left(\nu_\mu \rightarrow \nu_e ; L\right) \approx \sin^2 \left(2\theta_{12}\right) \sin^2 \left(1.27\Delta m^2_{21} \frac{L}{E}\right) \]

**Medium \(\Delta m^2\):**

- uncertainty in \(L/E\) corresponds to a fraction of an oscillation.
- \(1.27\Delta m^2 L/E \sim \pi/2\) is possible for some of the detected events

\(P\) (limit) determined by experiment corresponds to smallest \(\sin^2(2\theta)\) when \(1.27\Delta m^2 L/E = \pi/2\).
Even more on those confusing plots...

\[ P(\nu_\mu \rightarrow \nu_e; L) \approx \sin^2 (2\theta_{12}) \sin^2 \left( 1.27 \Delta m_{21}^2 \frac{L}{E} \right) \]

**Small \( \Delta m^2 \):**

- uncertainty in \( L/E \) corresponds to a fraction of an oscillation.
- \( \sin^2(1.27\Delta m^2 L/E) < 1 \) since \( L/E \) is always too small

\( P \) (limit) determined by experiment corresponds to larger and larger \( \sin^2(2\theta) \) as \( \Delta m^2 L/E \) shrinks.

(\textbf{Green} curve is contour of fixed probability to observe oscillation)
What could be happening?

• Nothing
• Two- or three-flavor oscillations $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$
• Oscillations into "sterile" neutrinos (ν’s just "disappear")
• Neutrinos decay (into what??)
• Matter-enhanced (Mikheyev-Smirnov-Wolfenstein mechanism) oscillations
• Extra dimensions
• Something else
Is anything happening?

1. Atmospheric neutrinos
   - cosmic ray interactions in the atmosphere produce ν’s through decays of π, K, μ.
   - expect ν_μ/ν_e ratio for upwards- and downwards-going neutrinos to be equal if no oscillations.
   - expect ν_μ/ν_e ratio for upwards-going neutrinos to shrink relative to downwards-going if ν_μ oscillates into ν_τ or ν_sterile but ν_e doesn’t.
   - Super-Kamikande finds ν_μ(up)/ν_μ(down) = 0.52 ± 0.05 but ν_e(up)/ν_e(down) ~ 1.
   - suggestive of ν_μ → ν_τ oscillations
Is anything happening?

2. Solar neutrinos...
Is anything happening?

... Solar neutrinos...

---

Is anything happening?

... Solar neutrinos...

Not enough neutrinos.

MSW mechanism?

(more on this next week)

<table>
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<tr>
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<th>$^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$ (SNU)</th>
<th>$^{71}\text{Ga} \rightarrow ^{74}\text{Ge}$ (SNU)</th>
<th>$^8\text{B}\nu$ flux (10$^9$cm$^{-2}$s$^{-1}$)</th>
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</table>

1 SNU (Solar Neutrino Unit) = 10$^{-38}$ captures per atom per second.

“3σ” errors.
Is anything happening?

3. LSND result

- 1 mA beam of 800 MeV (kinetic) energy protons produces $\pi^+$, $\mu^+$ which decay in flight (most $\pi^-$, $\mu^-$ captured in shielding)
- $\overline{\nu}_e / \overline{\nu}_\mu \approx 4 \times 10^{-4}$ in beam
- Look for $\overline{\nu}_e$ appearing in liquid scintillator volume
- find $82.8 \pm 23.7$ excess $\overline{\nu}_e$ events
- also see weaker evidence for $\nu_\mu \rightarrow \nu_e$
Is anything happening?

...LSND result

BNL E776

LSND has the only appearance result.
Next week

More about the existing results and planned experiments.