In obtaining the expression (11) the mass difference between the charged and neutral has been ignored. \(^{12}\)M. Ademollo and R. Gatto, Nuovo Cimento \textbf{44A}, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters \textbf{17}, 888 (1966). \(^{13}\)The predicted ratio [eq. (12)] from the current algebra is slightly larger than that \((0.23\%\) \) obtained from the \(\rho\)-dominance model of Ref. 2. This seems to be true also in the other case of the ratio \(\Gamma(\theta\to\pi^+\pi^-\pi^-)/\Gamma(\theta\gamma)\) calculated in Refs. 12 and 14. \(^{14}\)L. M. Brown and P. Singer, Phys. Rev. Letters \textbf{8}, 460 (1962).

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**A MODEL OF LEPTONS**

Steven Weinberg

Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

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Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons. \(^{2}\) This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields. \(^{3}\) The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

\[
L = \left[ \begin{array}{c}
\nu_e \\
\bar{e}
\end{array} \right]
\]  

\[
\mathcal{L} = -\frac{1}{4} \left( \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g A_{\mu} \times A_{\nu} \right)^2 - \frac{1}{4} (\partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu})^2 - \bar{R} \gamma^\mu (\partial_{\mu} - ig^* B_{\mu}) R - \bar{Y} \gamma^\mu (\partial_{\mu} - ig^* B_{\mu}) Y - \frac{1}{2} g^* B_{\mu} L 
\]  

\[
- \frac{3}{2} |\partial_{\mu} \phi - ig A_{\mu} \cdot \mathbf{T} \phi | + \frac{3}{2} g^* B_{\mu} \phi |^2 - G (\bar{L} \phi R + \bar{R} \phi \dagger L) - M_1^2 \phi \phi + h (\phi \phi^\dagger)^2. 
\]  

We have chosen the phase of the \( R \) field to make \( G \) real, and can also adjust the phase of the \( L \) and \( Q \) fields to make the vacuum expectation value \( \lambda = \langle \phi \rangle \) real. The "physical" \( \phi \) fields are then \( \phi^- \)
and
\[ \varphi_1 = (0 + \varphi - 2\lambda)/\sqrt{2}, \quad \varphi_2 = (0 - \varphi - 2\lambda)/i\sqrt{2}. \] (5)

The condition that \( \varphi_1 \) have zero vacuum expectation value to all orders of perturbation theory tells us that \( \lambda^2 \cong M_1^2/2\hbar \), and therefore the field \( \varphi_1 \) has mass \( M_1 \) while \( \varphi_2 \) and \( \varphi^- \) have mass zero. But we can easily see that the Goldstone bosons represented by \( \varphi_2 \) and \( \varphi^- \) have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates \( \varphi^- \) and \( \varphi_2 \) everywhere without changing anything else. We will see that \( G_e \) is very small, and in any case \( M_1 \) might be very large,\(^7\) so the \( \varphi_1 \) couplings will also be disregarded in the following.

The effect of all this is just to replace \( \varphi \) everywhere by its vacuum expectation value
\[ \langle \varphi \rangle = \lambda^1_0. \] (6)

The first four terms in \( \mathcal{L} \) remain intact, while the rest of the Lagrangian becomes
\[ -\frac{1}{2} \lambda^2 g^2 [ (A_{\mu}^1)^2 + (A_{\mu}^2)^2 ] - \frac{1}{2} \lambda^3 g (A_{\mu}^3 + g B_{\mu}) e^2 - \lambda G e e. \] (7)

We see immediately that the electron mass is \( \lambda G e \). The charged spin-1 field is
\[ \gamma_{\mu} = 2^{-1/2} (A_{\mu}^1 + i A_{\mu}^2) \] (8)
and has mass
\[ M_W = \frac{1}{2} \lambda G. \] (9)

The neutral spin-1 fields of definite mass are
\[ Z_{\mu} = (g^2 + g'^2)^{-1/2} (g A_{\mu}^3 + g' B_{\mu}), \] (10)
\[ A_{\mu} = (g^2 + g'^2)^{-1/2} (g' A_{\mu}^3 + g B_{\mu}). \] (11)

Their masses are
\[ M_Z = \frac{1}{2} (g^2 + g'^2)^{1/2}, \] (12)
\[ M_A = 0, \] (13)
so \( A_{\mu} \) is to be identified as the photon field.

The interaction between leptons and spin-1 mesons is
\[ \frac{ig}{2\sqrt{2}} \gamma_{\mu} (1 + \gamma_5)^\nu W_{\mu} \psi \text{ H.c.} \left[ \frac{igg'}{(g^2 + g'^2)^{1/2}} \gamma_{\mu} e A_{\mu} \right. \]
\[ + \left. \frac{1}{4} (g^2 + g'^2)^{1/2} \left[ \left( \frac{3g^2 - g'^2}{g^2 + g'^2} \right) \gamma_\nu e + \gamma_\nu \gamma_5 e - \gamma_\nu \gamma_5 e + \gamma_\nu \right] Z_{\mu} \right]. \] (14)

We see that the rationalized electric charge is
\[ e = gg'/\sqrt{(g^2 + g'^2)^{1/2}} \] (15)
and, assuming that \( W_{\mu} \) couples as usual to hadrons and muons, the usual coupling constant of weak interactions is given by
\[ G_W = \sqrt{2} = g^2/8M_W^2 = 1/2\lambda^2. \] (16)

Note that then the \( e - \varphi \) coupling constant is
\[ G_e = M_e/\lambda = 2^{1/4} M_e G_W^{1/2} = 2.07 \times 10^{-6}. \]

The coupling of \( \varphi_1 \) to muons is stronger by a factor \( M_1/M_e \), but still very weak. Note also that (14) gives \( g \) and \( g' \) larger than \( e \), so (16) tells us that \( M_W \sim 40 \text{ BeV} \), while (12) gives \( M_Z \sim M_W \) and \( M_Z \sim 80 \text{ BeV} \).

The only unequivocal new predictions made by this model have to do with the couplings of the neutral intermediate meson \( Z_{\mu} \). If \( Z_{\mu} \) does not couple to hadrons then the best place to look for effects of \( Z_{\mu} \) is in electron-neutron scattering. Applying a Fierz transformation to the \( W \)-exchange terms, the total effective \( e - \nu \) interaction is
\[ G_W \frac{\gamma_{\mu}}{\sqrt{2}} (1 + \gamma_5) \gamma_{\nu} (3g^2 - g'^2)/(g^2 + g'^2)^{3/2} \gamma_{\mu} e + \frac{3}{2} \gamma_{\nu} \gamma_e \gamma_5 e. \] (15)

If \( g \gg e \) then \( g \ll g' \), and this is just the usual \( e - \nu \) scattering matrix element times an extra factor \( \frac{3}{2} \). If \( g \approx e \) then \( g \approx g' \), and the vector interaction is multiplied by a factor \( -\frac{1}{2} \) rather than \( \frac{3}{2} \). Of course our model has too many arbitrary features for these predictions to be
SPECTRAL-FUNCTION SUM RULES, $\omega-\varphi$ MIXING, AND LEPTON-PAIR DECYES OF VECTOR MESONS*  

R. J. Oakes†  
Brookhaven National Laboratory, Upton, New York  
and  
J. J. Sakurai  
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics,  
The University of Chicago, Chicago, Illinois  

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Within the framework of vector-meson dominance, the current-mixing model is shown to be the only theory of $\omega-\varphi$ mixing consistent with Weinberg's first sum rule as applied to the vector-current spectral functions. Relations among the leptonic decay rates of $\rho^0$, $\omega$, and $\varphi$ are derived, and other related processes are discussed.

We begin by considering Weinberg's first sum rule\footnote{For a similar argument applied to the $\sigma$ meson, see Weinberg, Ref. 6.} extended to the $(1 + 8)$ vector currents of the eightfold way\footnote{R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1957).}:  

$$ \int dm^2 m^{-2} \rho_{\alpha \beta}^{(1)} (m^2) + \rho_{\alpha \beta}^{(0)} (m^2) = S_{\alpha \beta} + S'_{\alpha \beta} + \delta_{\alpha \beta} \omega^0 $$  

\footnote{See particularly T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). A similar phenomenon occurs in the strong interactions; the $\rho$-meson mass in zeroth-order perturbation theory is just the bare mass, while the $\omega$ meson picks up an extra contribution from the spontaneous breaking of chiral symmetry. See S. Weinberg, Phys. Rev. Letters 18, 507 (1967), especially footnote 7; J. Schwinger, Phys. Letters 24B, 473 (1967); S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967), Eq. (13) et seq.}


\footnote{S. Glashow, Nucl. Phys. 22, 579 (1961); the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions.}

\footnote{The history of attempts to unify weak and electromagnetic interactions is very long, and will not be reviewed here. Possibly the earliest reference is E. Fermi, Z. Physik 88, 161 (1934). A model similar to ours was discussed by S. Glashow, Nucl. Phys. 22, 579 (1961); the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions.}

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\footnote{On leave from the University of California, Berkeley, California.}

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\footnote{C. N. Yang, Phys. Rev. 98, 101 (1955).}

\footnote{This is the same sort of transformation as that which eliminates the nonderivative $\bar{\tau}$ couplings in the $\sigma$ model; see S. Weinberg, Phys. Rev. Letters 18, 188 (1967). The $\bar{\tau}$ reappears with derivative coupling because the strong-interaction Lagrangian is not invariant under chiral gauge transformation.}