

## Electron Transport Line for Mu2e Calibration System

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### Abstract

The electron production source for a linac-based calibration system for the Mu2e experiment will likely be above ground. Thus, a transport line is required to direct electrons to the Mu2e apparatus below ground. We simulate electrons passing through three vacuum tubes and two dipole magnets with constant magnetic field, ignoring fringe effects. The variations in incident position and angle of beam electrons are similar to those expected from a linac with normalized emittance like that of the AØ photoinjector. Seismic motion of magnets is included in the simulation. We find that most electrons will not reach the end of the transport line without additional focusing.

### Simulation parameters

Simulation was done using MATLAB. Electrons from the source are passed through a vacuum pipe, the first bending dipole magnet, a second vacuum pipe, the second bending magnet, then finally through a vacuum tube to the exit port. The vacuum pipes are 20, 10, and 30 meters long, respectively. The dipole bending magnets are modeled as cubes with constant 1 Tesla magnetic field and no fringe field. They will typically bend a 105 MeV electron through 90°. A thousand electron trajectories are calculated for each simulation run.

The transport line coordinate system has its  $x$  and  $y$  axes parallel to the ground and its  $z$ -axis orthogonal to the ground. The transport line elements are aligned with the  $x$  and  $z$  axes while the envelope of initial electron trajectories is centered on the  $x$ -axis.

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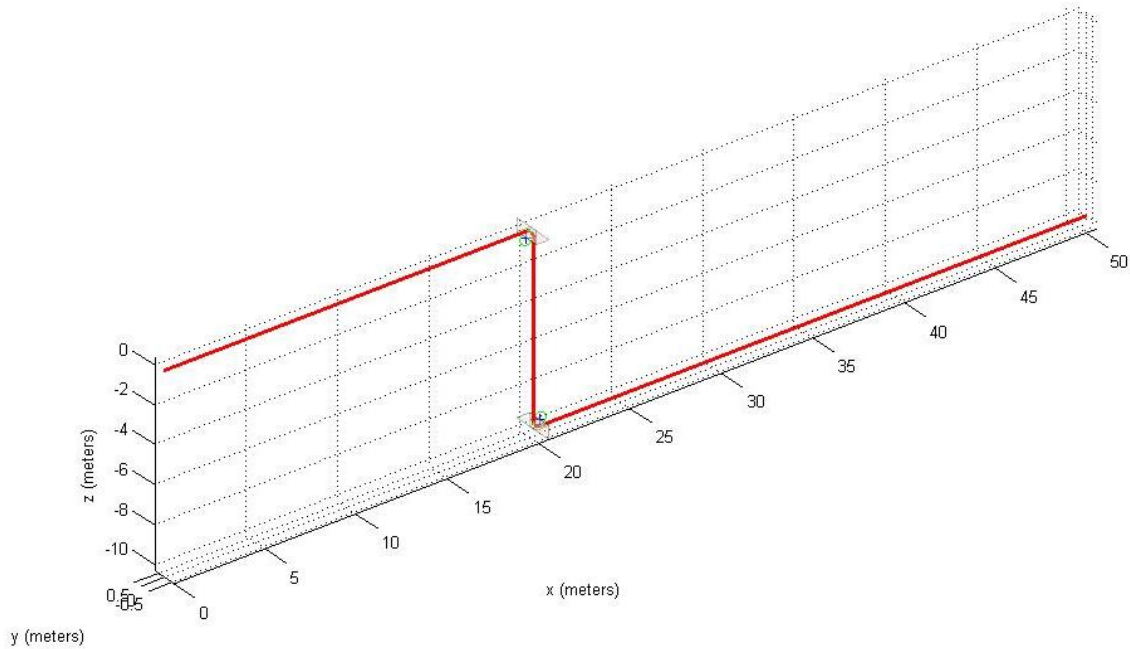


Figure 1: Ideal electron trajectory

The electrons' paths in the vacuum pipes are assumed to be straight lines. Their trajectories in the magnetic fields are calculated analytically as sections of helices.

The simulated electrons all have energy 105 MeV. Based on the AØ photoinjector parameters for normalized emittance, electrons were assumed to have an angular spread of  $\sigma_\theta = 53.5 \mu\text{rad}$ . The initial  $y$  and  $z$  coordinates of an electron are drawn from a Gaussian distribution with a standard deviation of 1 mm for each coordinate as shown in Figure 1. All electrons begin with  $x = 0$ .

Uncorrelated seismic motion of the magnets was simulated by translating the position of the magnets in  $x$ ,  $y$ , and  $z$  with a standard deviation of 1 micron.

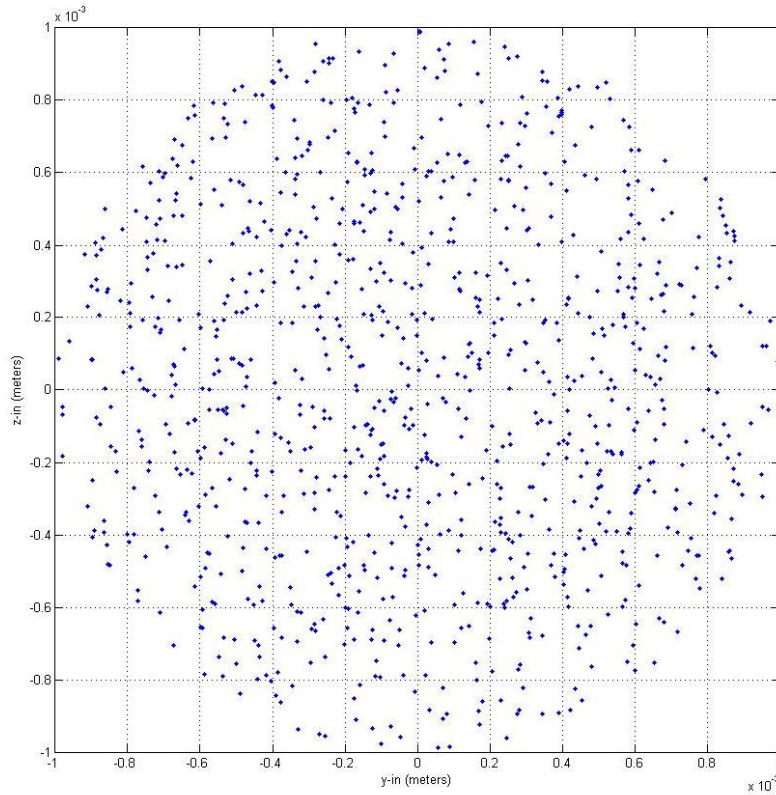


Figure 2: Initial electron  $y$  and  $z$  positions.

### Simulation results

Electrons were found to deviate significantly from the ideal path, as can be seen in Figure 3. Final vertical positions ranged from -2.38 meters to -18.75 meters, with mean -9.99 meters and standard deviation 2.04 meters. Ideal final vertical position is  $z = -10$  meters. Variation in the final  $y$ -coordinate was not as large as in  $z$ , ranging from -0.016 to 0.012 meters with standard deviation 0.0034 and mean  $-1.41 \times 10^{-4}$  meters. Ideal final  $y$  position is  $y = 0$ .

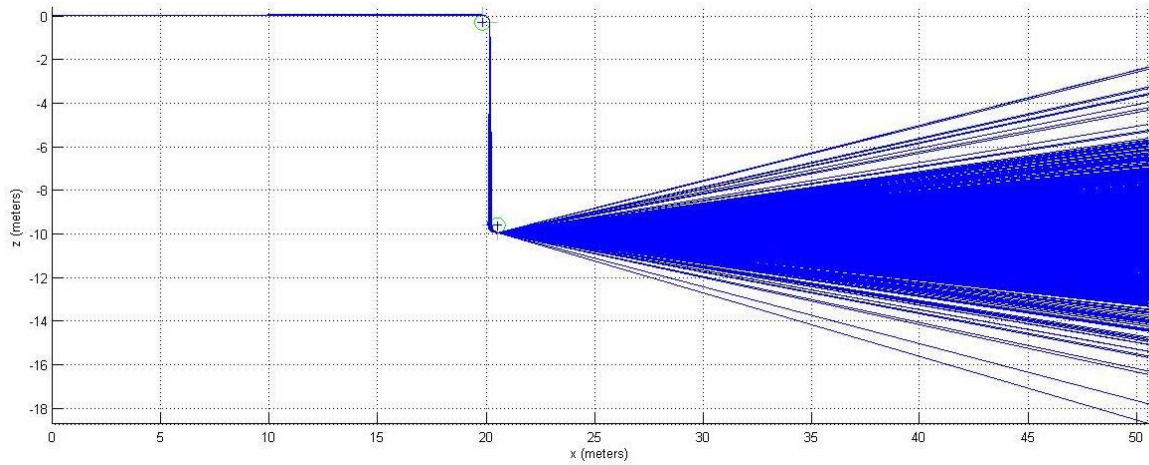


Figure 3: 1000 simulated electron tracks

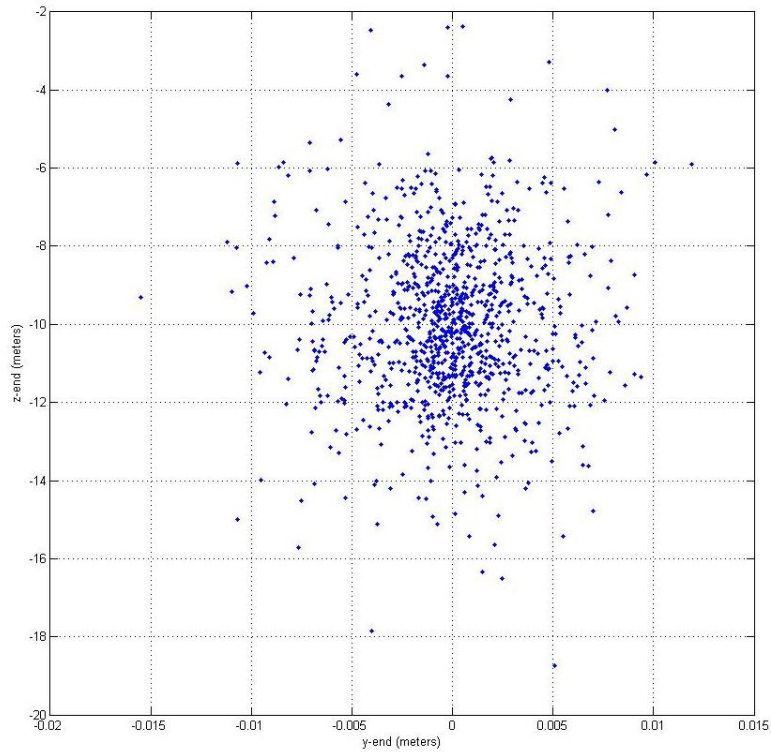


Figure 4: Electron y and z final positions. Note the very different horizontal and vertical scales.

The angular deviations of the electron's trajectory are amplified greatly by the dipole magnets. The randomized input angle is show in Figure 5. Angles ranged from -0.14292 milliradians to 0.13619 milliradians.

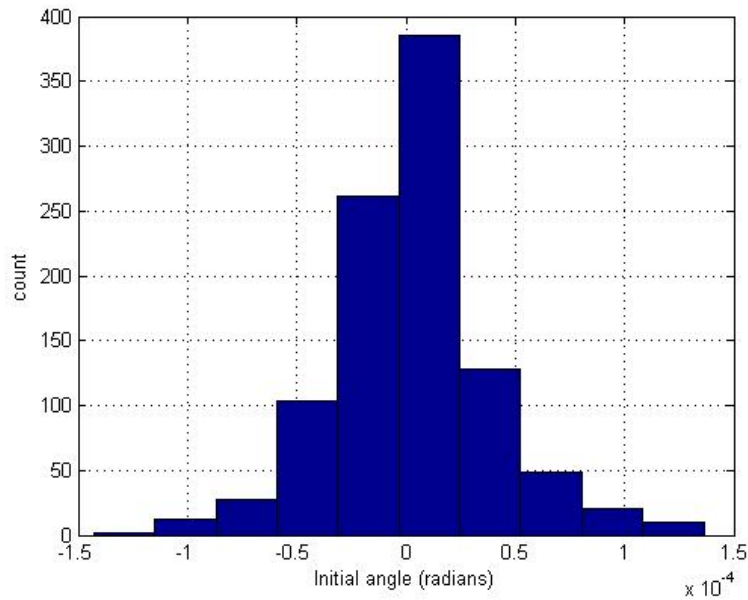


Figure 5: Distribution of input angles

The distribution of angles after the electrons pass through both magnets is shown in Figure 6.

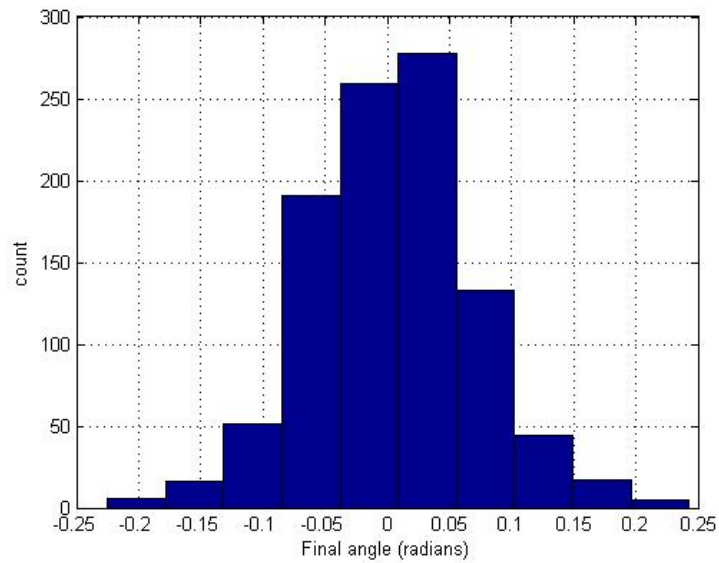


Figure 6: Distribution of output angles

Final angles ranged from -0.23 to 0.25 radians, or on the order of a few thousand times the initial angular variation. The dipole focusing that occurs is shown in Figure 7. Electrons entering the magnetic field with trajectories that are angled away from the exit plane of the magnet spend more time in the field and have their path bent more, while

those electrons angled toward the exit plane will be bent less. The result is a wide spread of directions coming out of the magnet.

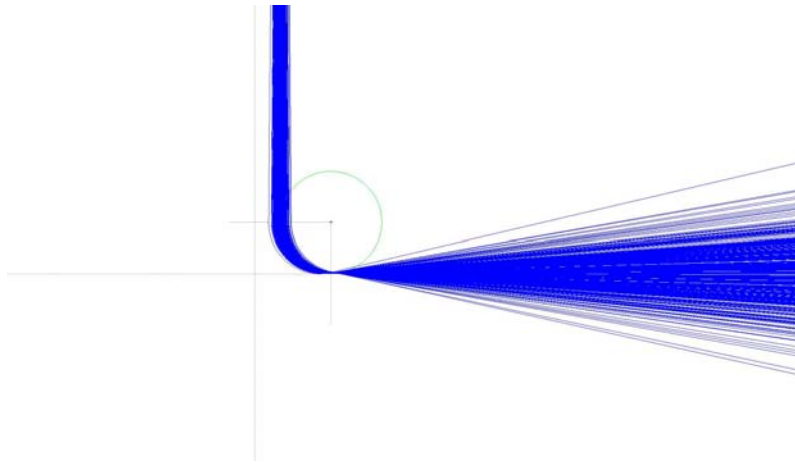


Figure 7: Dipole focusing in the second magnet

### Synchrotron radiation

Some rough calculations on the effect of synchrotron radiation were also done. The power loss is given by<sup>2</sup>

$$P = \frac{\mu_0 q^4 \gamma^4 a^2}{6\pi c}.$$

The relativistic acceleration is

$$a = \gamma^2 \frac{v^2}{r}$$

where the radius  $r$  is found using the Lorentz force law in a constant magnetic field:

$$p = qBr, \quad r = \frac{p}{qB}.$$

Substituting these values into the power loss equation gives

$$P = \frac{\mu_0 q^4 \gamma^4 B^2 \beta^4 c^3}{6\pi p^2}.$$

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<sup>2</sup> Chapter 16, Rohlf, James William, *Modern Physics from A to Z0*, Wiley, 1994

For a field of 1 T and an electron energy of 105 MeV, the power loss is 4.182 GeV/s. The radius of the circular path the electron takes while in the magnetic field is 35.02 cm. Assuming the magnet bends the electron through one quarter-turn, the electron is under centripetal acceleration for

$$t = \left( \frac{2\pi r}{4} \right) (\beta c)^{-1} = 1.835 \text{ ns},$$

giving a total power loss of

$$(4.182 \text{ GeV/s}) (1.835 \text{ ns}) = 7.675 \text{ eV}.$$

Note that the electron's fractional energy loss is less than  $10^{-7}$  and may be ignored safely in beamline simulations. The photon energy<sup>3</sup> is on the order of

$$E_t \equiv \pi \gamma^3 \hbar \omega_0$$

where  $\omega_0$  is the instantaneous angular frequency of the circular motion of an electron in a constant magnetic field. For a 105 MeV electron,  $\omega_0 = 855.96 \text{ MHz}$ , and  $E_t = 0.36 \text{ eV}$ . For an electron moving through a quarter-turn in a constant 1 Tesla field of radius 35 cm,  $7.675 \text{ eV} / 0.36 \text{ eV} \approx 21$  photons will be emitted, with variance  $\sim \sqrt{21} \sim 4.5$  photons.

## Conclusions

Although the transport line geometry used in the simulation is unlikely to be the same as that of the final design, some trends are apparent. For each dipole bending magnet the electron passes through, the angular variation is amplified greatly. The distance between the calibration linac and the Mu2e apparatus is likely to be tens of meters, and an electron that is propagated through a transport line built without focusing magnets will rarely make it to the line's exit port.

The largest contribution to electron displacement from the desired path is from the initial angular variation. With all other parameters set to zero variation so that only the initial trajectory angle changes, final  $z$  positions of the electron range from -3.88 m to -16.09 m. The next largest contribution in displacement is from the initial position. With all electrons initially traveling parallel to the  $x$ -axis, the final  $z$  coordinate ranged from -7.74 m to -12.28 m (Figure 8). With only magnet shifts taken into account, the final range of  $z$  positions was -10.01 m to -9.99 m. Position displacement due only to magnet shifts are much smaller compared to the incident position variation.

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<sup>3</sup> p. 280, Edwards, D.A., Syphers, M.J., *An Introduction To The Physics of High Energy Accelerators*, Wiley, 1993

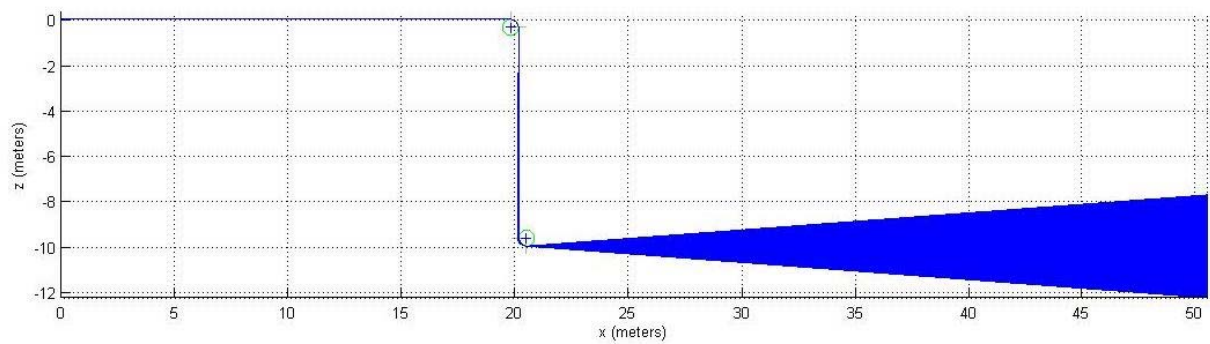


Figure 8: Electron trajectories with only initial position variation

Assuming the desired precision at the injection port is a few millimeters, focusing magnets will be required in the beamline.