

PROJECT DESCRIPTION

Stability, Matching and Sensitivity of Flat Beams

Personnel and Institution(s) requesting funding

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Project Overview

The International Linear Collider (ILC) is expected to begin a new era of discovery in elementary particles and high energy physics. To make this new advanced accelerator a reality, many critical R&D needs must be addressed. One of them is the in-depth understanding, and control, of the dynamics of the flat beam that the ILC damping rings are designed to produce. This advanced accelerator will produce beams with an extremely large ratio of horizontal to vertical emittance. In this unprecedented regime of operation, the beam is likely to be extremely sensitive to changes in physical and lattice parameters. Therefore, a tight and robust control of the flat beam is a must to guarantee stability and achieve the desired beam quality and flatness.

Our *long-term goal* is to develop a research line on advanced control of particle beams in accelerators. The *overall objective* of this proposal, which is the next step toward attaining our long-term goal, is to determine stability conditions, develop adaptive tuning techniques, and define control strategies to optimize transport and increase robustness against uncertainties and hardware imperfections, for extremely flat beams. The *rationale* that underlines this research work is that the results of the stability study for flat beams will contribute to define an optimal lattice for ILC, and that the development of adaptive and robust control techniques will allow easy tuning for low emittance and accurate matching.

We are well positioned to undertake the proposed research because we have assembled a research team (PI, graduate students, collaborators) that uniquely combines the diverse range of expertise (stability analysis, nonlinear adaptive control, optimization techniques, optics, beam dynamics) that is needed to reach a definitive outcome. In addition, a partnership with the University of Maryland Electronic Ring (UMER) has been established for experimental testing of the theoretical results obtained at Lehigh University. The PI, with 15 years of higher education, has been trained in all the disciplines involved in this multidisciplinary project. In addition, the proposed research is strongly supported by prior work that validates the proposed approach and by preliminary results obtained by the PI in the area of beam matching in particle accelerators.

We expect to achieve the objective of this proposal by pursuing the following specific aims:

- *Analytical and numerical stability analysis of flat beams.* The linear stability properties of mismatch perturbations around the matched beam envelope has been recently analyzed by Lund and Bukh [1] using the linearized Kapchinskij-Vladimirskij (KV) equations [2]. The analysis was carried out for continuous focusing, periodic solenoidal, and periodic quadrupole transport lattices assuming equal emittances for both the x and y transverse directions ($\epsilon_x = \epsilon_y = \epsilon$). This is an extension of previous work on this topic by Struckmeier and Reiser [3]. Hofmann [4] studied the linear stability of anisotropic beams with space charge based on self-consistent Vlasov-Poisson equations, where the case of distinct emittances ($\epsilon_x \neq \epsilon_y$) was considered. However, the analysis for this important case was carried out only for continuous focusing transport lattices. We plan on extending this prior work by studying stability of matched solutions for periodic focusing transport lattices and ratios of horizontal to vertical emittance characteristic of ILC. Both FODO and TME lattices will be considered. When possible, an analytic approach using averaging theory [5] and periodically switched systems theory [6] will be considered. Otherwise, numerical analysis will be used to map regions of parametric instability as functions of ϵ_x/ϵ_y .
- *Development of adaptive and robust control techniques for matching solutions for flat beams.* A multi-parameter extremum seeking scheme has been implemented, and successfully tested in simulations, for the tuning of the lens strengths in matching channels [7, 8]. The scheme can be used for real-time optimization taking advantage of its non-model-based nature, which represents an advantage with respect to other model-based optimization techniques such as nonlinear and dynamic programming. To accelerate convergence, a hybrid scheme is envisioned where the optimal lens strengths are computed off-line using extremum seeking or another optimization technique, and used as initial conditions for the lens strengths for an on-line extremum seeking controller. Under this framework, the extremum seeking algorithm will be playing the role of a non-model-based adaptive controller, which is one of its unique characteristics, that ensures a well-matched beam at the end of the matching channel independently of the changes in the system and lattice parameters. Adaptive control is extremely valuable for flat beams whose quality is extremely sensitive to these changes and errors. We plan on extending the results obtained so far for round isotropic ($\epsilon_x = \epsilon_y$) beams to flat anisotropic ($\epsilon_x \neq \epsilon_y$) beams.
- *Sensitivity analysis of matched solutions.* Beams with severe aspect ratio, like the ones considered in this proposal, are likely to be extremely sensitive to lattice variations and errors. It is therefore crucial to assess the impact of such variations and errors on the beam quality and flatness. A sensitivity analysis of matched solutions will be carried out for:
 1. gradient errors
 2. tilt errors (skew)

Although numerical analysis will be considered as a tool for sensitivity assessment, it is the goal of this proposal to determine analytical expressions of the beam sensitivity that can be used not only to assess the impact of variations and errors on the beam quality and flatness but also to be incorporated into the cost function used by the extremum seeking procedure. The inclusion of a figure of sensitivity into the extremum seeking cost function allows the optimization procedure to converge to the least sensitive solution among all the possible matching solutions in an undetermined system.

- *Experimental validation.* Matching of flat beams using extremum seeking will be demonstrated experimentally at the University of Maryland Electronic Ring (UMER) in a typical Collins insertion and other matching schemes. This experiment, although different, is similar in essence to the Ring-to-Linac-Transfer-Line (RTLTL) in ICL in the sense that both require the matching of flat beams. Even though the experiment will be performed using UMER FODO lattices, we expect that the results will be extendable to ILC TME lattices since the extremum seeking controller is a non-model-based controller (its effectiveness does not depend on the “model” of the actuator). Theoretical predictions for sensitivity and stability will be also tested experimentally.

The research is *innovative* because the unique approach proposed in this proposal has not been applied before to flat beam stabilization and control. As *outcomes of this research project*, we expect to define stability requirements for flat beams that help to define an optimal lattice for ILC. In addition, we expect to develop adaptive and robust control techniques for tuning and matching of flat beams, minimizing at the same time their sensitivity to lattice and actuators (lenses) imperfections. Experimental benchmarking of stability results and control strategies will be carried out in collaboration with Institute for Research in Electronics and Applied Physics, University of Maryland, College Park (see attached letter of support). The research proposed is *significant* because its outcomes are expected to contribute to one of the most important R&D requirements of ILC.

Broader Impact

The proposed research will *advance discovery while promoting teaching and learning*. Integration of this research project with education will be accomplished through motivation and improved teaching of undergraduate and graduate students. The project seeks two educational objectives. The first focuses on offering a new course on “Accelerator Optics and Beam Dynamics” for senior undergraduate students and graduate students in Physics and Engineering at Lehigh University. By improving and integrating undergraduate teaching, we expect to create a “magnet” for undergraduate students that will serve to attract them to the graduate program. The second focuses on educating graduate students in this multidisciplinary area, who will in turn help to achieve the first objective by collaborating in undergraduate teaching and presenting their research work at departmental seminars. As part of the education of these graduate students, they will be encouraged to participate in professional meetings and conferences. As a member of a minority group (hispanic) himself, the PI expects to be a role model for young people from underrepresented groups who want to develop careers in engineering research. He will encourage and reward the *participation of underrepresented groups* in his research projects. The *infrastructure for research and learning will be enhanced* in the Department by establishing a partnership with the Institute for Research in Electronics and Applied Physics, University of Maryland, College Park. Outcomes of the proposed research will be *disseminated broadly*. Not only will results be published in peer-reviewed journals and the proceedings of prestigious and widely attended conferences, but all the results of the project will be posted at the PI’s personal web site. Finally, the research is expected to *benefit society as a whole* by addressing some of the critical R&D needs of the International Linear Collider, and therefore making possible the construction of an experimental facility that is expected to begin a new era of discovery.

Results of Prior Research

The PI has already invested a great deal of effort into the problems of beam stability and beam matching in particle accelerators using non-model-based optimal adaptive control techniques [7, 8].

Extremum Seeking. Extremum seeking control [9], a popular tool in control applications in the 1940-50's, has seen a resurgence in popularity as a *real time optimization* tool in different fields of engineering. In addition to being an optimization method, extremum seeking is a method of *adaptive control*, usable both for tuning set points in regulation and optimization problems and for tuning parameters of control laws. It is a *non-model based* method of adaptive control, and, as such, it solves, in a rigorous and practical way, some of the same problems as neural network and other intelligent control techniques. Aerospace and propulsion problems (formation flight [10], combustion instabilities [11, 12], flow control [13], compressor rotating stall [14]), automotive problems (anti-lock braking, engine mapping), bioreactors[15], and charged particle accelerators [7, 8] are among its applications. Extremum seeking is applicable in situations where there is a nonlinearity in the control problem, and the nonlinearity has a local minimum or a maximum which defines a desired operating point of the system. The nonlinearity may be in the plant (physical system), as a physical nonlinearity, possibly manifesting itself through an equilibrium map. Hence, one can use extremum seeking for tuning a set point to achieve an optimal value of the output. The parameter space can be multivariable.

The discrete-time implementation [16] is depicted in Figure 1, where q denotes the Z -transform variable. The high-pass filter is designed with $0 < h < 1$, and the modulation frequency ω is selected such that $\omega = \alpha\pi, 0 < |\alpha| < 1$, and α is rational. Without loss of generality, the static nonlinear block $J(\theta)$ is assumed to have a minimum J^* at $\theta = \theta^*$, the desired operating point. The extremum seeking procedure guarantees that the estimation of θ , denoted as $\hat{\theta}$ in the figure, will converge to θ^* , minimizing J . We give next an elementary intuitive explanation about how the scheme works [9]. The perturbation signal $a \sin(\omega k)$ fed into the plant helps to get a measure of gradient information of the map $J(\theta)$. We posit $J(\theta)$ of the form:

$$J(\theta) = J^* + \frac{J''}{2}(\theta - \theta^*)^2 \quad (1)$$

where $J'' > 0$. Any twice continuously differentiable function $J(\theta)$ with a local minimum can be approximated locally in this way. The assumption $J'' > 0$ is made without loss of generality. If $J'' < 0$, i.e., J has a local maximum, we just replace $\gamma(\gamma > 0)$ in Figure 1 with $-\gamma$. The purpose of the algorithm is to make $\theta - \theta^*$ as small as possible, so that the output $J(\theta)$ is driven to its minimum J^* . We start by noting that $\hat{\theta}$ in Figure 1 denotes the estimate of the unknown optimal input θ^* . Let $\tilde{\theta} = \theta^* - \hat{\theta}$ denote the estimation error. Thus,

$$\theta(k) - \theta^* = \hat{\theta}(k) + a \sin(\omega k) - \theta^* = a \sin(\omega k) - \tilde{\theta}(k) \quad (2)$$

which, when substituted into the expression for $J(\theta)$, gives

$$J(k) = J(\theta(k)) = J^* + \frac{J''}{2}(a \sin(\omega k) - \tilde{\theta}(k))^2 \quad (3)$$

Expanding this expression further, and applying the basic trigonometric identity $2 \sin^2(\omega k) = 1 - \cos(2\omega k)$, one gets

$$J(k) = J(\theta(k)) = J^* + \frac{a^2 J''}{4} + \frac{J''}{2} \tilde{\theta}^2(k) - a J'' \tilde{\theta}(k) \sin(\omega k) - \frac{a^2 J''}{4} \cos(2\omega k) \quad (4)$$

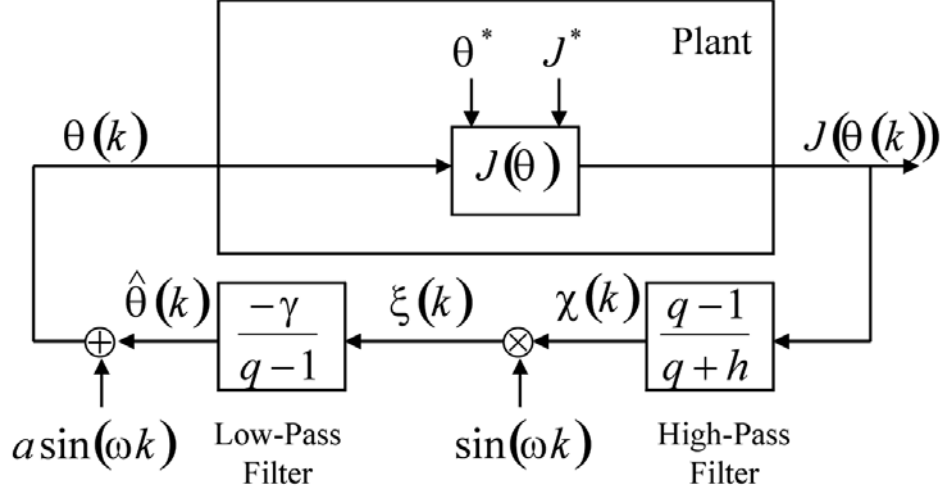


Figure 1: Extremum seeking control scheme

The high pass filter $(q-1)/(q+h)$ applied to the output, serves to remove the DC components, namely,

$$\chi(k) = \frac{J''}{2} \tilde{\theta}^2(k) - aJ'' \tilde{\theta}(k) \sin(\omega k) - \frac{a^2 J''}{4} \cos(2\omega k) \quad (5)$$

This signal is then “demodulated” by multiplication with $\sin(\omega k)$, giving

$$\xi(k) = \frac{J''}{2} \tilde{\theta}^2(k) \sin(\omega k) - aJ'' \tilde{\theta}(k) \sin^2(\omega k) - \frac{a^2 J''}{4} \cos(2\omega k) \sin(\omega k) \quad (6)$$

Applying again $2\sin^2(\omega k) = 1 - \cos(2\omega k)$, as well as the identity $2\cos(2\omega k)\sin(\omega k) = \sin(3\omega k) - \sin(\omega k)$, we arrive at

$$\xi(k) = -\frac{aJ''}{2} \tilde{\theta} + \frac{aJ''}{2} \tilde{\theta}(k) \cos(2\omega k) + \frac{a^2 J''}{8} (\sin(\omega k) - \sin(3\omega k)) + \frac{J''}{2} \tilde{\theta}^2(k) \sin(\omega k) \quad (7)$$

First, we neglect the last term because it is quadratic in $\tilde{\theta}$ and we are interested only in local analysis. Second, when we pass the signal through the low-pass filter $-\gamma/(q-1)$, the high frequency components (last two terms) get greatly attenuated. Thus, we neglect them, getting

$$\hat{\theta}(k) = \frac{\gamma}{q-1} \frac{aJ''}{2} \tilde{\theta}(k) \iff \hat{\theta}(k+1) - \hat{\theta}(k) = \frac{\gamma a J''}{2} \tilde{\theta}(k) \quad (8)$$

Recalling that $\tilde{\theta} = \theta^* - \hat{\theta}$, and noting that θ^* is constant, we can conclude that $\hat{\theta}(k+1) - \hat{\theta}(k) = -(\tilde{\theta}(k+1) - \tilde{\theta}(k))$, and we can write

$$\tilde{\theta}(k+1) = \tilde{\theta}(k) - \frac{\gamma a J''}{2} \tilde{\theta}(k) = (1 - \frac{\gamma a J''}{2}) \tilde{\theta}(k) \quad (9)$$

Since $\gamma a J'' > 0$, we can make $|1 - \frac{\gamma a J''}{2}| < 1$, thus making this system stable. We conclude that $\tilde{\theta} \rightarrow 0$, or, in terms of the original problem, $\hat{\theta}$ converges to within a small distance of θ^* .

Beam Matching. Let the z coordinate represent the position along the design beam trajectory, which we assume varies between 0 and L , and thus the xy plane is the transverse plane for the particle beam. At each z location, let $X(z)$ and $Y(z)$ represent the semi-axes of the beam envelope in the x and y planes, respectively. We are given initial conditions for the beam at $z = 0$, the transport system's entrance location. These conditions characterize the beam coming from the preceding section of the transport or accelerator system. They may be translated into initial conditions for the beam envelopes in the x plane (X_{ini}, X'_{ini}) and in the y plane (Y_{ini}, Y'_{ini}), where the prime indicates differentiation with respect to z . In matching systems we are also given desired final conditions, or target conditions, at $z = L$, the exit location of the matching channel. We denote this target conditions as (X_{tar}, X'_{tar}) and (Y_{tar}, Y'_{tar}) . They are prescribed by the acceptance requirements of the next section of the transport or accelerator system.

Denoting $x = [X \quad X' \quad Y \quad Y']^T$, we define

$$x_{ini} = x(0) = \begin{bmatrix} X_{ini} \\ X'_{ini} \\ Y_{ini} \\ Y'_{ini} \end{bmatrix}, \quad x_{fin} = x(L) = \begin{bmatrix} X_{fin} \\ X'_{fin} \\ Y_{fin} \\ Y'_{fin} \end{bmatrix}. \quad (10)$$

In addition, we define a target value for x at the exit of the matching channel denoted as $x_{tar} = [X_{tar} \quad X'_{tar} \quad Y_{tar} \quad Y'_{tar}]^T$, and desired beam profiles (beam trajectories) for $X(z)$ and $Y(z)$ denoted as $X_{des}(z)$ and $Y_{des}(z)$ respectively. Given $x_{ini}, x_{tar}, X_{des}(z)$ and $Y_{des}(z)$, we use an extremum seeking procedure to minimize the cost function J given, for instance, by

$$J = \{k_1 J_1 + k_2 J_2 + k_3 J_3\}^{\frac{1}{2}} \quad (11)$$

$$J_1 = K_X (X_{final} - X_{target})^2 + K_Y (Y_{final} - Y_{target})^2 \quad (12)$$

$$J_2 = K_{dX} (X'_{final} - X'_{target})^2 + K_{dY} (Y'_{final} - Y'_{target})^2 \quad (13)$$

$$J_3 = \int_0^L w(z) \left[K_{iX} (X(z) - X_{des}(z))^2 + K_{iY} (Y(z) - Y_{des}(z))^2 \right] dz, \quad (14)$$

where $K_X, K_Y, K_{dX}, K_{dY}, K_{iX}$, and K_{iY} are weight constants, and w_z is an integral weight function. Minimizing J , we make $X_{final} \rightarrow X_{target}, Y_{final} \rightarrow Y_{target}, X'_{final} \rightarrow X'_{target}, Y'_{final} \rightarrow Y'_{target}, X_z \rightarrow X_{des}(z)$, and $Y_z \rightarrow Y_{des}(z)$. It is important to emphasize that the cost function can be arbitrarily defined. Cost function (11) is only one example. For instance, if measure of the derivatives is not available, we can replace the J_2 component of the cost function by a new J_1 -like component where target values for X and Y are defined for a different position $z \neq L$. If a J_3 -like component is included in the cost function, the designer is required to have an intuitive understanding as to what makes a good desired trajectory. The beam envelope will track the desired trajectory as closely as possible. These conditions leads to optimality only if the desired trajectory is chosen properly (in an optimal sense). The choice of the desired trajectory is particularly important for under-determined systems where the number of lenses is strictly higher than four. In these cases the solution for the matching problem (i.e., making $x_{fin} = x_{tar}$) is not unique and the choice of the desired trajectory has a decisive influence on the outcome of the optimization procedure.

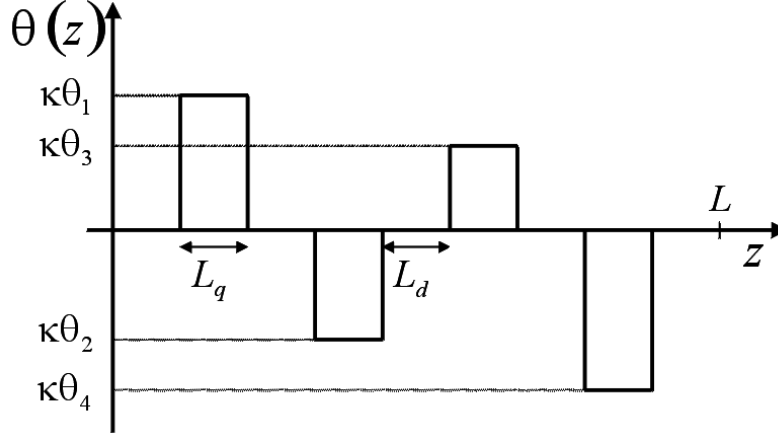


Figure 2: Focusing function

The problem is formulated as finite-“time” optimal control ($0 \leq z \leq L$), with bang-bang controls (on-off controls) of fixed durations but varying intensities (i.e., with a very coarse discretization in “time” which results in a highly constrained waveform for the control θ (focusing strength) as it is shown, for instance, in Figure 2 (for the four lenses, hard-edge approximation case), for a plant that is nonlinear. This is far from being a standard optimization problem. To add complexity to the problem, we are seeking robustness against uncertainties of the system for a successful practical implementation of the control method.

The extremum seeking procedure has been proved effective for round isotropic beams. A simulation study has been carried out modeling the beam dynamics using the KV coupled-envelope equations [2],

$$X'' - \theta(z)X - \frac{2K}{X+Y} - \frac{\epsilon_X^2}{X^3} = 0 \quad (15)$$

$$Y'' + \theta(z)Y - \frac{2K}{X+Y} - \frac{\epsilon_Y^2}{Y^3} = 0 \quad (16)$$

The function $\theta(z)$ is the focusing (control) function. K is the beam perveance, $\epsilon_X = \epsilon_Y = \epsilon$ are the effective emittances of the beam in the x and y planes, respectively. An example of the focusing function $\theta(z)$, based on a hard-edge approximation, is shown in Figure 2 for a matching channel of four quadrupoles, and can be written as

$$\theta(z) = \begin{cases} \kappa\theta_1 & z \in [L_d, L_d + L_q] \\ \kappa\theta_2 & z \in [2L_d + L_q, 2L_d + 2L_q] \\ \kappa\theta_3 & z \in [3L_d + 2L_q, 3L_d + 3L_q] \\ \kappa\theta_4 & z \in [4L_d + 3L_q, 4L_d + 4L_q] \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

where κ is a constant, L_d is the drift length, and L_q is the quadrupole lens length. The matching channel parameters θ_1 , θ_2 , θ_3 , and θ_4 for the examples shown below must satisfy in addition the following constraints:

$$\begin{aligned} 0 &\leq \theta_1 \leq 50 \\ -50 &\leq \theta_2 \leq 0 \\ 0 &\leq \theta_3 \leq 50 \\ -50 &\leq \theta_4 \leq 0. \end{aligned} \quad (18)$$

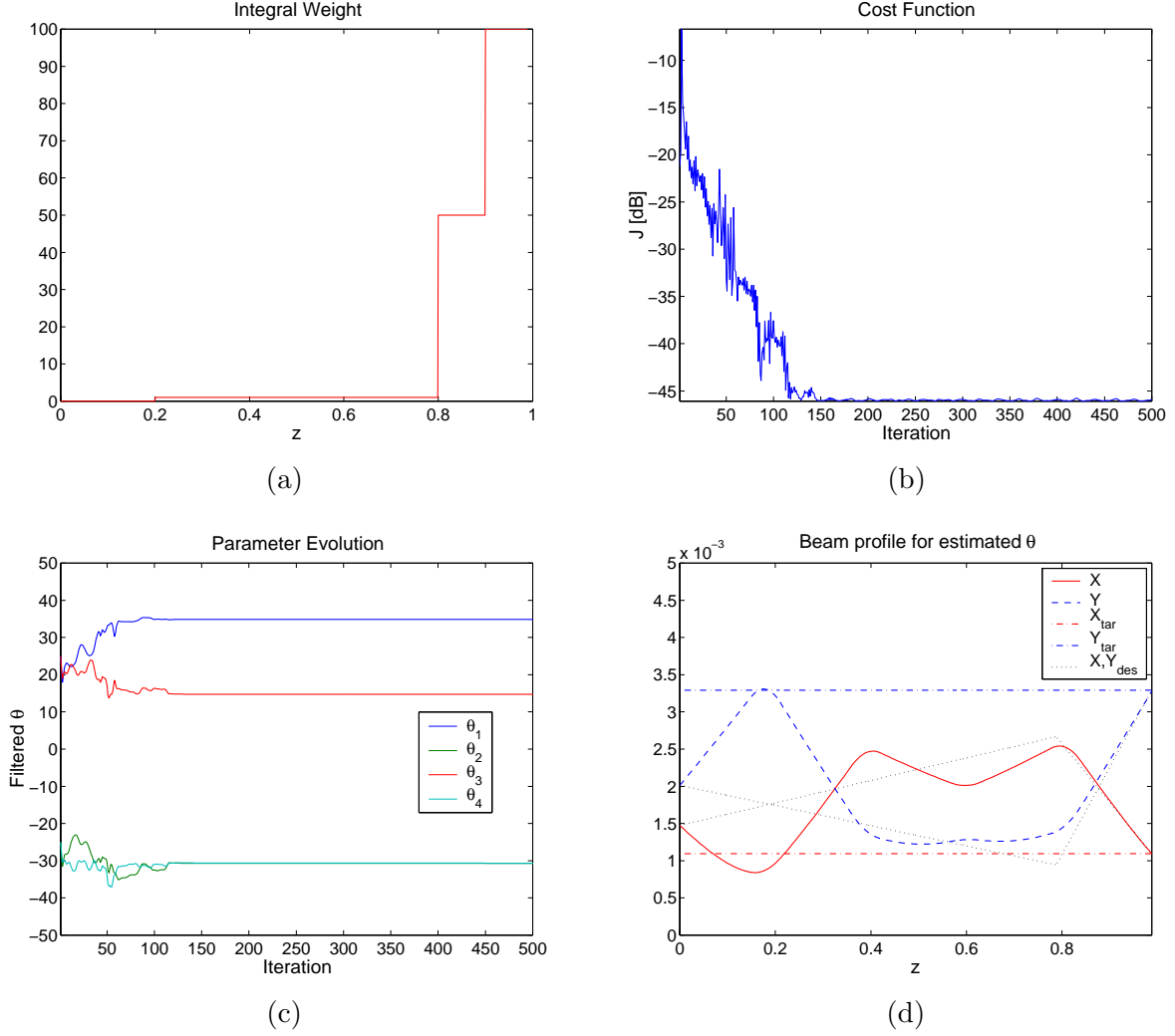


Figure 3: Integral weight (a), cost function (b), θ evolution (c), Beam profile for θ_{final}

Figure 3 shows the simulation results for a matching channel of four quadrupoles. In this case, the three components of the cost function (11) are considered. We take $X_{des}(z)$ and $Y_{des}(z)$ as a combination of two linear functions as shown in Figure 3-d (dotted line). By defining desired trajectories for X and Y , we try not only to match the target conditions at the end of the matching channel but also to reduce excursions inside the channel. The integral weight $w(z)$ is shown in Figures 3-a. The slope of the last section of the desired beam profile coincides with the target conditions for the derivatives in order to facilitate their matching. The use of only one linear function, connecting X_{ini} and Y_{ini} , with X_{tar} and Y_{tar} respectively, would be in conflict with the terminal conditions for the derivatives. The time evolution of $\theta_1, \theta_2, \theta_3, \theta_4$ in Figure 3-c shows that a steady value is reached after 150 iterations. This can be also noted from Figure 3-b, where the cost function does reach a steady value after 150 iterations, showing a very fast convergence. Figure 3-d shows the beam profile (in m) for θ_{final} , the value of θ after convergence. Not only the matching of the target conditions is very good, but also the matching of the desired profile. This is explained by how the cost function was defined.

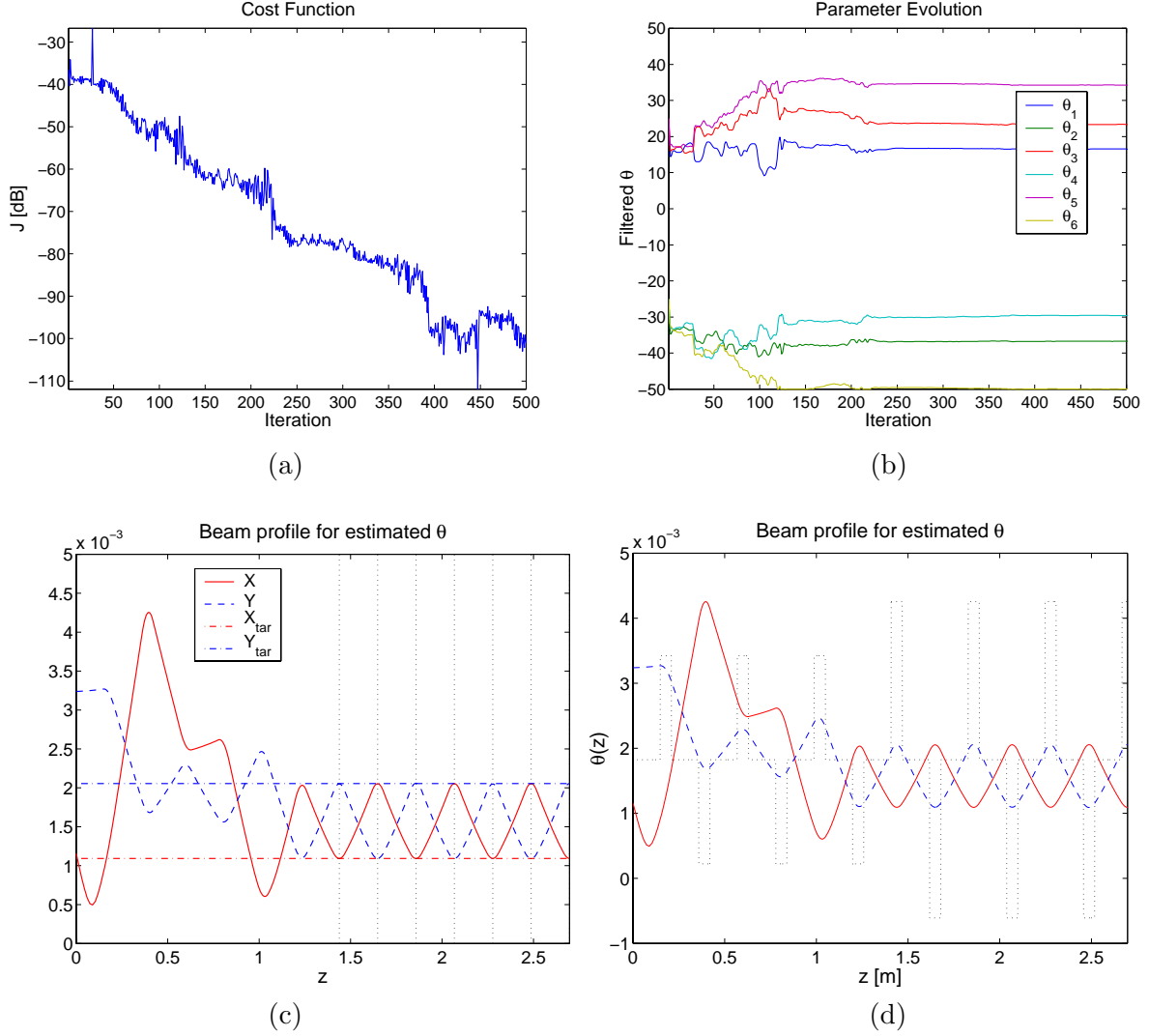


Figure 4: Cost function evolution (a), θ evolution (b), beam profile for θ_{final} (c),(d)

Figure 4 shows the simulation results for a matching channel of six quadrupoles followed by a periodic channel. In this case, only J_1 -like components of the cost function (11) are considered. However, target values are defined not only at the end of the matching section but at several positions within the periodic channel, indicated by vertical dotted lines in Figure 4-c. The time evolution of $\theta_1, \theta_2, \theta_3, \theta_4$ in Figure 4-b shows that a steady value is reached after 400 iterations. However, the estimation for θ is already acceptable after 250 iterations. This can be also noted from Figure 4-a, where the cost function does reach a steady value after 400 iterations, showing a very fast convergence. Figure 4-c shows the beam profile (in m) for θ_{final} , the value of θ after convergence. The matching of the target conditions at all the prescribed positions within the periodic channel is very good. Figure 4-d shows the focusing function $\theta(z)$, without scale, plotted over the beam profile (in m) for θ_{final} . The strength of the first six quadrupoles (matching section) are varied by the extremum seeking algorithm, as it is shown in Figure 4-b, to match the target conditions defined within the periodic channel. The strengths of the quadrupoles within the periodic channel are kept constant. Figure 5 shows the evolution of the beam profile as a function of the iteration number.

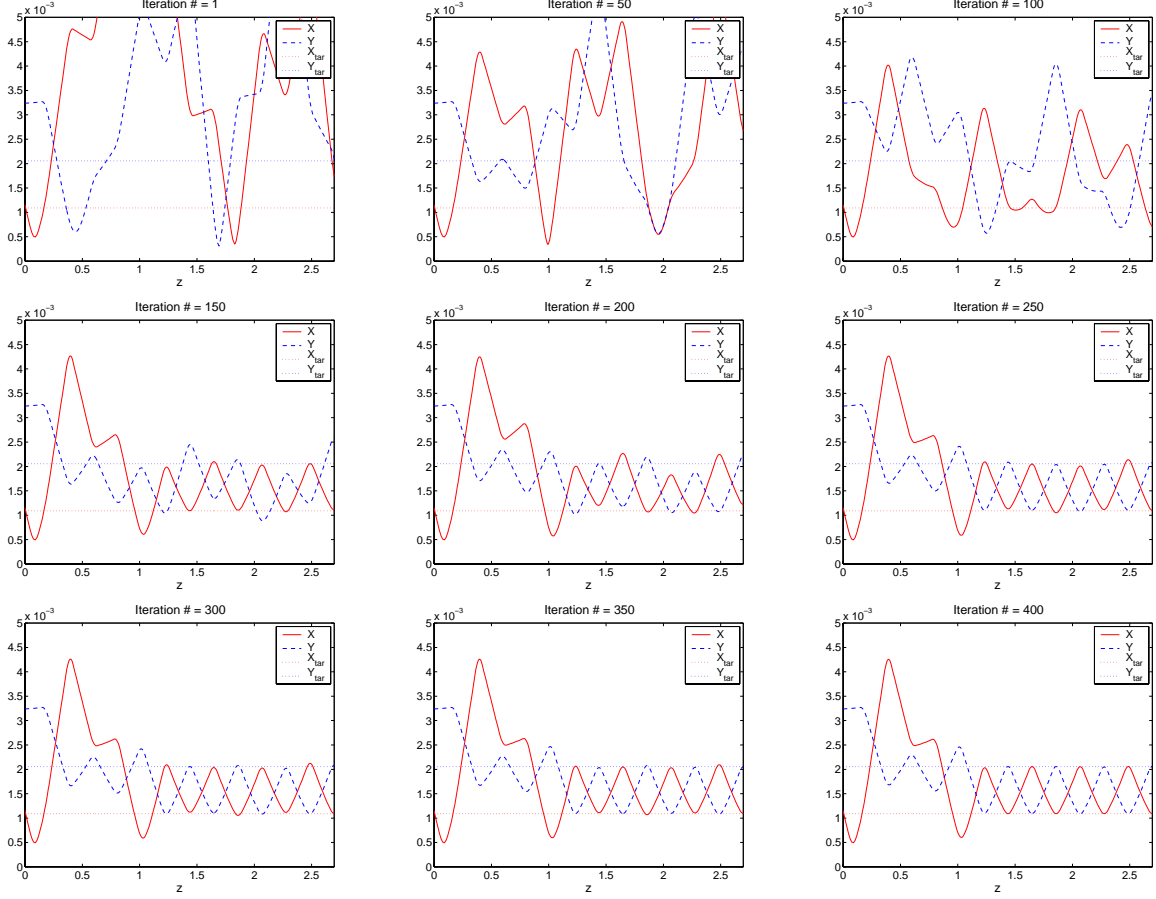


Figure 5: Evolution of the beam profile

Stability of matched envelope solutions. The transverse evolution of the envelope beam in a linear transport channel can be modeled by the Kapchinskij-Vladimirskij (KV) envelope equations [2],

$$\begin{aligned} X'' + \kappa_x(s)X - \frac{2K}{X+Y} - \frac{\epsilon_x^2}{X^3} &= 0, \\ Y'' + \kappa_y(s)Y - \frac{2K}{X+Y} - \frac{\epsilon_y^2}{Y^3} &= 0. \end{aligned} \quad (19)$$

where the prime indicates differentiation with respect to s (the s coordinate represent the position along the design trajectory). At each location s , $X(s)$ and $Y(s)$ represent the semi-axes of the beam envelope in the transverse x and y planes, respectively. The functions $\kappa_x(s)$ and $\kappa_y(s)$ are the focusing (control) functions. K is the beam perveance, ϵ_x and ϵ_y are the effective emittances of the beam in the x and y planes, respectively. We are interested in studying the stability of the matched solution (X_0, Y_0) . The nonlinear stability of the nonlinear non-autonomous system (19) can be studied numerical but an analytical study is still pending. Linear stability of the matched solution can be carried out by writing $X(s) = X_0(s) + x(s)$, $Y(s) = Y_0(s) + y(s)$, and linearizing the nonlinear non-autonomous system (19) around the matched solution. A linear time variant (LTV) system is obtained to

describe the dynamics of the perturbation variables $x(s), y(s)$,

$$\begin{aligned} x'' + \left(\kappa_x + \frac{3\epsilon^2}{X_0^4} + \frac{2K}{(X_0+Y_0)^2} \right) x + \frac{2K}{(X_0+Y_0)^2} y &= 0 \\ y'' + \left(\kappa_y + \frac{3\epsilon^2}{X_0^4} + \frac{2K}{(X_0+Y_0)^2} \right) y + \frac{2K}{(X_0+Y_0)^2} x &= 0. \end{aligned} \quad (20)$$

Writing $\zeta = (x, y, x', y')^T$, we can rewrite the LTV model (20) as $\dot{\zeta}(s) = A(s)\zeta(s)$, where,

$$A(s) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_1(s) & -a_0(s) & 0 & 0 \\ -a_0(s) & -a_2(s) & 0 & 0 \end{pmatrix}, \quad \begin{aligned} a_0(s) &= \frac{2K}{(X_0(s)+Y_0(s))^2}, \\ a_1(s) &= \kappa_x(s) + \frac{3\epsilon^2}{X_0^4(s)} + a_0(s), \\ a_2(s) &= \kappa_y(s) + \frac{3\epsilon^2}{Y_0^4(s)} + a_0(s); \end{aligned} \quad (21)$$

In a periodic lattice of period S , the LTV system (20) has also periodicity S . In order to get the evolutionary behavior of the LTV periodic system, we have to determine the so-called state transition matrix $\Phi(s, s_0)$, which satisfies,

$$\frac{d\Phi(s, s_0)}{ds} = A(s)\Phi(s, s_0), \quad \Phi(s_0, s_0) = I. \quad (22)$$

Floquet theorem states that the LTV system (20) is exponentially stable if and only if $\Phi(S, 0)$ is *Schur*, that is, all eigenvalues of $\Phi(S, 0)$ have magnitudes less than one. Current approaches compute $\Phi(S, 0)$ numerically, assuming $\epsilon_x = \epsilon_y$, for periodic lattices and infer stability properties from it. The PI and his group is actively working towards obtaining general analytical conditions for stability and/or instability, which include the $\epsilon_x \neq \epsilon_y$ case, without making use of the *smooth approximation*. The availability of closed-form analytical conditions would simplify the stability analysis of matched solutions in particle accelerators with periodic lattices.

For linear time invariant (LTI) systems, the transition matrix can be computed as $\Phi(s, s_0) = \exp(A(s - s_0))$. Unfortunately, this is not the case for LTV systems because $A(s)$ and $\int_0^s A(\tau)d\tau$ do not usually commute, i.e., $[A(s), \int_0^s A(\tau)d\tau] = A(s) \int_0^s A(\tau)d\tau - \int_0^s A(\tau)d\tau \cdot A(s) \neq 0$. A series expression called *Peano-Backer Formulae* can be used to write down the transition matrix for LTV systems, $\Phi(s, s_0)$,

$$\Phi(s, s_0) = I + \int_{s_0}^s A(\tau_1)d\tau_1 + \int_{s_0}^s A(\tau_1) \left(\int_{s_0}^{\tau_1} A(\tau_2)d\tau_2 \right) d\tau_1 + \dots, \quad (23)$$

The *Peano-Backer Formulae* converges uniformly with respect to s and s_0 . Indeed, since the matrices are locally bounded, for each interval H there exists a constant C such that the spectrum norm $\|A(s)\| < C$ for all $\tau \in H$. Thus the k -term in the series is bounded by $C^k(t - t_0)^k/k!$. Therefore, we can approximately write down $\Phi(S, 0)$, with $\eta = CS$ small, as

$$\Phi(S, 0) = I + \int_0^S A(s)ds + O(\eta/2) \equiv I + A_{av}S + O(\eta/2) \approx I + A_{av}S \quad (24)$$

where,

$$SA_{av} = \begin{pmatrix} 0 & 0 & S & 0 \\ 0 & 0 & 0 & S \\ -\int_0^S a_1(s)ds & -\int_0^S a_0(s)ds & 0 & 0 \\ -\int_0^S a_0(s)ds & -\int_0^S a_2(s)ds & 0 & 0 \end{pmatrix}. \quad (25)$$

The LTV periodic system (20) is exponentially stable if and only if $I + A_{av}S$ is *Schur*, or equivalently, if and only if $A_{av}S$ is *Hurwitz*, that is, all the eigenvalues of $A_{av}S$ have negative real parts. Similar result can be obtained using averaging theory [5] or periodically switched systems theory [6]. Writing A_{av} as a function of, or a bound in terms of, the average value of the matched solution and the physical parameters $(K, \epsilon_x, \epsilon_y)$, would allow to use the *Routh-Hurwitz Theorem* to infer closed-form stability conditions for the matched solution. The theorem gives necessary and sufficient conditions for stability of the system matrix A_{av} in terms of the coefficient of its characteristic equation $|\lambda I - A_{av}|$, which are in turn functions of the average value of the matched solution and the physical parameters $(K, \epsilon_x, \epsilon_y)$.

Prior NSF/DOE Support. None.

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Facilities, Equipment and Other Resources

The facilities and resources at Lehigh University are ideally suited for the proposed project. Professor Eugenio Schuster, leads the Laboratory for Control of Complex Systems in the Department of Mechanical Engineering and Mechanics at Lehigh University. The research group currently includes eight graduate students (3 Ph.D. students and 5 M.Sc. students). The laboratory is equipped with a large number of last-generation desktop computers running under Windows and Linux. Professor Eugenio Schuster and his group specialize in the application of advanced nonlinear control methods to complex physical systems.

Two major facilities, the Fairchild-Martindale Library and the Linderman Library, house the university collection of more than one million volumes. Both libraries provide collaborative learning spaces, wireless connectivity, and comfortable lounge areas. The Fairchild-Martindale Library contains books, journals, newspapers, audio-visual resources, and microform collections in all branches of science, engineering, mathematics, and the social sciences, including business and education. Subscriptions to more than 3,000 print journals and 3,000 electronic journals allow the university to compete successfully with many larger institutions in supporting scholarly research. The “virtual electronic library” at Lehigh is just as important as the print-based one. Lehigh has available a full range of electronic indexes, reference works, full text databases, and image databases, all of which are accessible to Lehigh staff and students from any computer on campus (including those in the residence facilities) or off campus via modem. A single Web-based interface allows the student to move seamlessly from Lehigh’s own online catalog (named ASA after Lehigh founder Asa Packer) to databases of citations, abstracts, articles or book reviews to the full text of many of these resources.

Library and Technology Services provides computing services to all university departments and research centers, serving the needs of students, faculty, and administrative users. More than 400 microcomputers (primarily IBM-compatible and some Apple personal computers) are distributed across campus for convenient use by Lehigh staff and students at more than 20 computing sites. Local and wide area networking solutions are in place to give students and faculty access to site-licensed software applications and central file space from the campus sites or their residence facility. The Fairchild-Martindale Computing Center houses a network of high-performance computers, configured as a centralized network service cluster. UNIX-based workstations and a new Beowulf cluster are available for research applications. The university computing capacity and bandwidth are constantly being increased to meet the escalating demand. Lehigh also offers higher-speed connections to the research-based Internet2 network by virtue of its charter membership in that organization.

First year Project Activities and Deliverables

- Stability analysis of matched solutions in periodic lattices (FODO and TME) using averaging theory and periodically switched systems theory in order to determine analytical conditions for stability and/or instability in terms of the parameters of the system.
- Study of the dependence of the stability properties on the geometry of the lattice. Criterion definition for optimal design of periodic lattice in ILC.
- Numerical or analytical determination of stability and/or instability region maps as functions of ϵ_x/ϵ_y . Ratios of horizontal to vertical emittance characteristic of ILC will be considered.
- Design and numerical testing of extremum-seeking-based controllers for adaptive tuning of lens strengths in FODO and TME lattices.

Second year Project Activities and Deliverables

- Sensitivity analysis of the matched solutions to different types of actuator errors and parameter changes.
- Incorporation of a sensitivity figure into the extremum seeking cost function with the ultimate goal of making the procedure converge to less sensitive solutions.
- Experimental matching of flat beams at UMER using extremum seeking controllers.
- Experimental validation at UMER of theoretical predictions for stability and sensitivity of flat beams.

Budget justification: Lehigh University

We propose a funding level for the research project of approximately \$68,000 per year over two years for a total request of \$136,841. The uniqueness of the research topics envisioned for this project make it highly appropriate for graduate student research. We propose to support 1 Ph.D. student in this research project. The work will be directed and supervised by the PI with the equivalent of 1 month salary support.

PERSONNEL.

1. Dr. Schuster will devote 8.33% effort over the year to the project. This effort will be paid as 1 month summer salary.
2. One graduate research assistant (GRA) will work on the project full time (50% effort during the academic year and 100% effort during the summer).

FRINGE BENEFITS.

Employee benefits are direct-charged as a percentage of salaries and wages at rates set by DOD/ONR Audit Office Negotiation Agreement dated October 3, 2005. The negotiated rate for FY 05/06 for full-time staff is 31.3% and 8.1% for part-time employees. The 8.1% rate is also applied to the graduate research assistant stipend during the three summer months.

TRAVEL.

The requested travel expenses will primarily support domestic travels of the PI and his graduate student to report results to the particle accelerator community at scientific conferences. The travel costs are estimated to be \$1,250 per trip per person (airfare \$350, \$500 per diem @ \$125/day for 4 days, \$400 registration).

Funds are requested for the graduate student to spend three months at University of Maryland Electronic Ring (UMER) for experimental tests during each year of the project. The travel expenses are estimated to be \$250 for airfare, and \$2,250 for lodging (short term rental) \$2,250). Daily transportation and meals will be covered from the student's stipend.

MATERIALS AND SUPPLIES.

Funds are requested for routine computer supplies and software upgrades.

OTHER DIRECT COSTS.

(1) Funds are requested for 20 credits of tuition each year for the full-time GRA. The tuition rate for academic year 05/06 has been set at \$970 per credit. *The university will provide a*

50% reduction in tuition in support of this proposal. An estimated increase of about 4% is applied for future years.

(2) Funds are requested for publication charges (\$1,000). The estimated publication charges are consistent with those of high quality journals, such as AIP and IEEE, and covers costs of color diagrams, etc.

(3) Funds are requested for postage, communications, and shipping (\$500 per Year). Communications charges are for long distance telephone charges to collaborate with colleagues regarding research findings, often in lieu of more expensive travel.

INDIRECT COSTS.

Indirect costs are charged as a percentage of modified total direct costs (MTDC) at a rate of 54% for FY 05/06, set by DOD/ONR Audit Office Negotiation Agreement on June 11, 2003. This rate is used for future years. Indirect costs are not charged on equipment, tuition and subcontract charges over \$25,000.

Two-year budget, in then-year K\$

Institution: Lehigh University

Item	First year	Second year	Total
Other Professionals	8,244	8,574	16,818
Graduate Students	19,200	19,968	39,168
Undergraduate Students	0	0	0
Total Salaries and Wages	27,444	28,542	55,986
Fringe Benefits	2,969	3,088	6,057
Total Salaries, Wages and Fringe Benefits	30,413	31,630	62,043
Equipment	0	0	0
Travel	5,000	5,000	10,000
Materials and Supplies	500	0	500
Other direct costs	11,588	11,992	23,580
Institution 2 subcontract	0	0	0
Total direct costs	47,501	48,622	96,123
Indirect costs(1)	20,203	20,590	40,793
Total direct and indirect costs	67,704	69,212	136,916



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January 5th, 2006

Professor Eugenio Schuster
Packard Laboratory, Room 550D
19 Memorial Drive West,
Lehigh University,
Bethlehem, PA 18015-3085


Dear Eugenio:

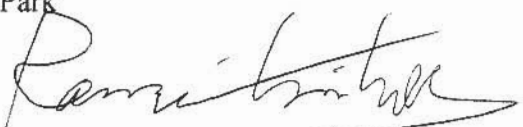
We were interested to learn of your proposal to NSF/DOE entitled "Stability, Matching and Sensitivity of Flat Beams." As beam control at the University of Maryland Electron Ring (UMER) is a major component of our effort in beam physics research, the methods that you and your colleagues pioneered using "Extremum Seeking" techniques are of great interest. Further, the work you propose to extend the theoretical study of beam stability to the case of flat beams with extreme emittance ratios, from the standpoint of modern approaches from control theory, is also worth pursuing and should benefit the accelerator community at large.

We commit to provide to you and your group at Lehigh University such collaborative support as necessary to effectively carry out the experimental part of the proposed program.

Please feel free to include this letter in your proposal package as evidence of our support and commitment.

Sincerely,


Patrick G. O'Shea,
Professor and Chair, Electrical and Computer Engineering Department,
University of Maryland, College Park

Rami A. Kishek
Associate Research Professor,
Institute for Research in Electronics and Applied Physics,
University of Maryland, College Park


Santiago Bernal
Assistant Research Scientist,
Institute for Research in Electronics and Applied Physics,
University of Maryland, College Park
