PROGRESS REPORT

Effects of CSR in Linear Collider Systems

Personnel and Institution(s) requesting funding

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Collaborating personnel; Not requesting funding here.

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Project Overview (Proposal written circa January 2004; only references have been updated)

I. Motivation

There are two points at which coherent synchrotron radiation (CSR) could be of concern in linear colliders. First, it may cause transverse x-emittance degradation in bunch compressors, since energy changes due to CSR get mapped into transverse coordinates through dispersion. Second, it might cause longitudinal bunch instabilities in the damping rings at high current, possibly leading to a quasi-periodic, sawtooth behavior of the bunch length. Damping ring designs contain many meters of wigglers (for instance 46 m at NLC and 432 at TESLA), to reduce the damping time to a manageable value. The coupling impedance from CSR in the wigglers, combined with that from bending magnets, could induce the feared instability. For a review of microbunch instabilities due to CSR see [1].

II. Bunch Compressors and Computation of CSR from Arbitrary Orbits

Preliminary estimates of emittance growth in bunch compressors have been made by Emma and Woodley [2]. They have done calculations for the Stage 1 and Stage 2 compressors (BC1 and BC2) of the NLC and for the Tesla bunch compressor (TBC) [3]. For BC1, BC2, and TBC their figures for emittance growth $\Delta\epsilon_x/\epsilon_x$ due to CSR were 2.4%, 5.5%, and 1%, respectively. Although these values are regarded as reasonably small compared to other sources of emittance growth, they are based on a simplified model of the CSR field, and cannot be considered a definitive conclusion. The field calculation, as implemented in M. Borland's code ELEGANT [4], treats the source of the radiation as a line charge. In a macroparticle simulation the transverse charge distribution is projected onto a longitudinal line to produce an effective radiation source. Moreover, the formula for the field is for a source moving on a trajectory which is straight except for a single bend. Consequently there is no proper integration of single bends to make a true chicane orbit. Actually, there should be a residual field after a bend which does not decay completely before the next bend is reached.

There exist large codes to compute CSR from an arbitrary charge/current distribution on fairly general orbits (by Dohlus [5] at DESY and Rui Li at JLAB [6]), but these have proved to be cumbersome for actual design work and are not much used for that purpose. We have devoted considerable effort to finding a better calculational method, intended to be useful both for bunch compressors and rings, and perhaps also for wigglers.

III. Ongoing and Proposed Work on Bunch Compressors and Computation of CSR from Arbitrary Orbits

IIIA.

One approach that we have explored in recent months is to make a Fourier analysis of fields and sources in all spatial dimensions, with account of the large-wavelength shielding of CSR by the vacuum chamber (in a perfectly conducting parallel plate model). This avoids the tricky integrations over singularities of the Green function that make the usual approach in space-time quite difficult. The price to pay is in dealing with fast oscillations of the integrand in the inverse Fourier transform to compute the field. We hoped that this problem could be handled by the method of stationary phase, but it turns out that this method is only partially effective. Since it still might be used in combination with other procedures, we are currently writing a careful report on our experience.

IIIB.

We have recently made what appears to be great progress by using a Fourier series only in the vertical coordinate Y, perpendicular to the plane of the orbit. This makes it trivial to satisfy the field boundary conditions on the parallel plates that model the vacuum chamber. The resulting 2D wave equation has a "mass" term, the mass being the vertical wave number, and a remarkably simple Green function with a softer singularity than the usual Green function for the 3D wave equation. We think that this should provide an efficient and simple numerical method, and propose the implementation of such a method as a main item of research. We will first study a chicane bunch compressor with the charge/current source coming from a bunch with energy chirp, evolving only in response to the fields of the dipoles. Horizontal transverse spread of the charge distribution is fully accounted for, whereas the vertical distribution is arbitrary but constant in time. Later we hope to make the calculation self-consistent, allowing the phase space distribution to be affected by CSR. This could be done in a macroparticle simulation, or preferably in a less noisy Vlasov treatment if that could be done in reasonable computation time.

IV. Damping Ring Instabilities

A representation of the impedance for CSR in wigglers has been proposed by Wu, Raubenheimer, and Stupakov (WRS)[7]. Wu, Stupakov, Raubenheimer and Huang (WSRH) [8] have combined this impedance with the usual impedance for CSR in dipoles to discuss the longitudinal instability threshold in damping ring designs for NLC and TESLA, and for the existing prototype damping ring at the KEK ATF. The instability study is done with coasting beam theory for a line-charge beam, and without shielding of CSR. The conclusion is that the instability is indeed worrisome. The threshold is close to the nominal current for NLC and ATF. On the other hand, the dipole and wiggler impedances scale differently with frequency, and that leads to a possibility of optimizing the damping ring design to raise the threshold, perhaps by a factor of 4.

V. Proposed Work on Damping Ring Instabilities

Although the work of WSRH is a valuable first survey of the problem, it involves some serious assumptions that one would like to avoid. We propose to pursue the following improvements:

1. Avoid coasting beam theory by applying our program for numerical integration of the nonlinear Vlasov-Fokker-Planck (VFP) equation [9, 10]. We would begin with the Haïssinski equilibrium distribution, and see whether it becomes unstable under time evolution by the VFP integrator. As the authors WSRH point out, the coasting beam theory is doubtful in this instance, because the Boussard criterion does not apply: the bunch length is not much larger than the unstable wave lengths.
2. Study possible saturation of any instability, again using the VFP code. That code has proved to be very useful for long term simulations, giving plausible results over several damping times.
3. Include shielding of CSR, which will be necessary in studying dynamics of unstable cases, and may have some effect on thresholds as well. We are not yet sure how to describe shielding for the wiggler radiation, and there is the complication that the vacuum chamber has different sizes in bends and wigglers. Some innovative approximations will certainly be necessary.
4. Criticize the mathematics and physics of the model of the wiggler impedance in WRS. This is for an infinite wiggler, and takes as its starting point results of Saldin *et al.* [11, 12]. As far as we know the complicated Saldin analysis has not been verified by other authors, and in any case there are some puzzling singularities in the result that we would like to understand. After becoming familiar with the problem, perhaps we can treat the case of a wiggler of finite length, which of course would be more relevant for the prediction of thresholds.
5. Try to include non-zero transverse extent of the bunch. This would relate to work on the 2D Green function mentioned above.

VI. Budget and Personnel

Warnock is a retired SLAC physicist with several years experience in CSR, see, for example,[1, 10]. The proposal is based on his recent work, our joint progress over the last few months, and discussions with other experts at SLAC. Ellison has some modest experience with radiation by moving charges from his work on channeling radiation at CERN and Aarhus in the late eighties and is now deeply involved in the basic issues of CSR from particle bunches on more or less arbitrary orbits. Bassi has been hired as a PostDoc, as of mid June 2003, after completing a Ph.D. at DESY and the University of Bologna. He is making a substantial contribution to our CSR work.

Warnock is not asking for financial support; he finds it sufficient to get theoretical and numerical collaboration from Ellison and Bassi. Ellison is not asking for financial support either as CSR is one item in the research of his current DOE grant - DE-FG03-99ER41104 for "Investigations of Beam Dynamics Issues at Current and Future Accelerators". In addition, Ellison has funds in his DOE grant to partially support the CSR work of Bassi. We are requesting \$36.8K/year for the 3 year period June 1, 2004 to May 31, 2007 to fill out the support of Bassi's CSR work and to partially support a graduate research assistant. The \$36.8K includes salary, fringe benefits, tuition, health insurance and the 50% indirect cost rate. Our current LCRD/DOE funding arrived too late to support a graduate student in the fall, however we have just hired a student who will begin work in January.

The proposal outlines an ambitious program that we find quite challenging.

Progress Report (January 2006)

A. Bunch Compressor and Computation of CSR from Arbitrary Planar Orbits

Considerable progress has been made since the proposal was written and so it is worthwhile to reframe our basic goal here. We study the influence of coherent synchrotron radiation (CSR) on particle bunches traveling on arbitrary planar orbits between parallel conducting plates which represent the vacuum chamber. Our ultimate goal is to follow the time evolution of the 4D phase space density (PSD) by solving the Vlasov-Maxwell equations in the time domain. This should provide simulations with lower numerical noise than a Monte Carlo approach, and allow one to study such issues as emittance degradation and microbunching due to CSR in bunch compressors. The fields excited by the bunch will be computed in the laboratory frame from a new formula that leads to much simpler computations than usual methods. The nonlinear Vlasov equation, formulated in the interaction picture, will be integrated in the beam frame by approximating the Perron-Frobenius (PF) operator. (The PF method is basically a method of local characteristics, [9]) In the process of developing a self-consistent CSR code we have found it important to do Monte Carlo studies at various levels. In addition, it is important to do as much analytic work as possible and to validate codes and methods by benchmarking.

Considerable progress has been made on the computation of CSR from arbitrary planar orbits as outlined in the above paragraph and as proposed in Section III above. Our work has been presented at the ICFA ERL2005 workshop, PAC2005 [13] and FEL2005. An outline of our Vlasov-Maxwell approach will be published shortly in NIMA [14]. As a result of Bassi's participation in the ERL workshop, he was asked to be the point person for an overview article on CSR codes which will be published shortly in NIMA [15]. We learned a great deal about the efforts of others in this exercise. Some of our work was discussed in an invited address by Warnock at the September COULOMB05 workshop in Senigallia, Italy [16] as well as in the Stupakov-Warnock contribution to the ICFA Beam Dynamics Newsletter 35 [1] edited by C. Biscari with a special theme section on coherent synchrotron radiation (CSR).

We discuss our progress in four parts: the field calculation, the lab to beam transformation, the Liouville-Maxwell Approximation(LMA) and the self-consistent Vlasov-Maxwell calculation. We have found that the calculations are subtle and thus it has been important to study the LMA as a step toward a self-consistent calculation. In addition, it is important to understand the degree to which the LMA is an approximation to the self-consistent case. At each stage of our development we have carefully studied possible sources of errors as well as the accuracy of the calculations. While our ultimate goal is the calculation of the phase space density defined by the Vlasov-Maxwell system, much of our work to date has followed a Monte Carlo approach to obtain a deep understanding of the self-consistent evolution of the charge density. This is needed as a guide to the phase space calculation, which is complicated by the difficulty of determining the support of the PSD.

A1. Field Calculation

A detailed report [17] on the approach based on the full Fourier method has been written as promised in IIIA. Our conclusion is that the method of stationary phase, applied to reduce the number of integrations and the field calculation time, is not a good approximation in a large enough region of Fourier space and thus the computation time is too long for practical use. Although a negative result, we gained a great deal of understanding and it was helpful in

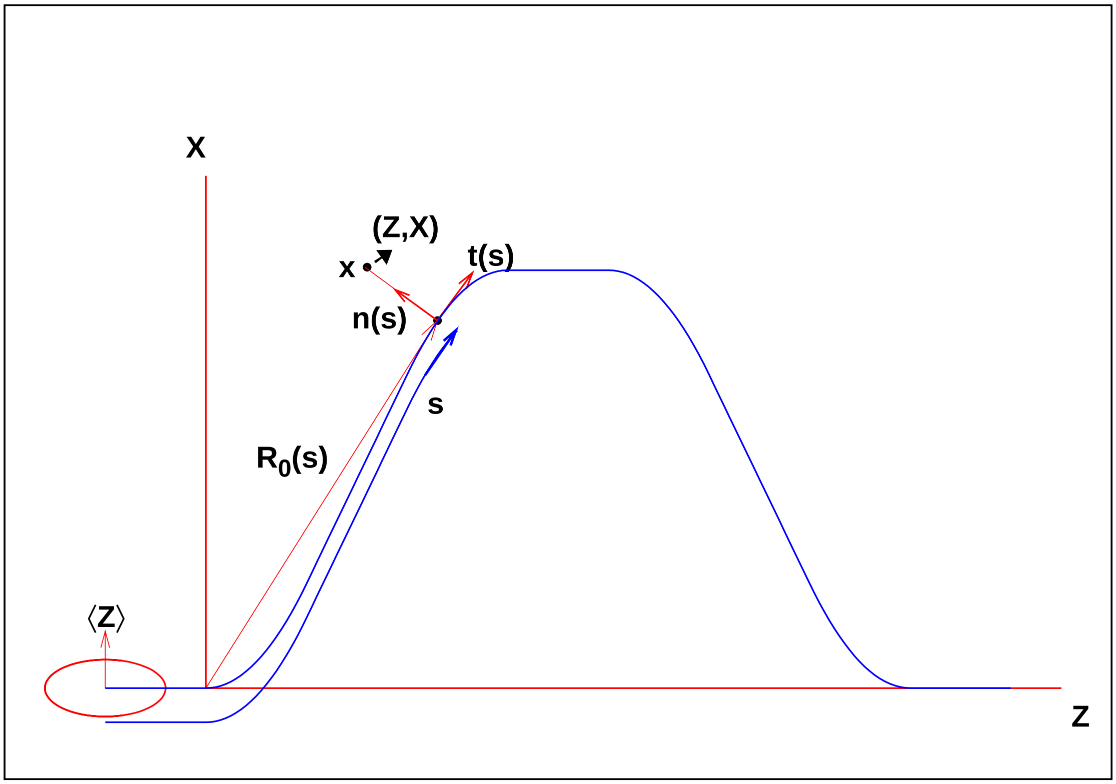

Figure 1: Lab frame (Z, X) and beam frame (s, x) for a chicane bunch compressor.

the implementation of the method based on the 2D wave equation of IIIB. Nevertheless, the method of stationary phase may be useful in combination with other procedures and might be effective in a third variant of the Fourier method (1D wave equation after a Fourier transform in x).

In IIIB above we proposed work on a new approach based on a 2D Klein-Gordon equation. Here, the fields excited by the bunch are computed in the laboratory frame from a new 2D integration formula that leads to much simpler computations than usual methods which use retarded or Lienard-Wiechert potentials. We have derived our formula in two ways one of which is presented in [14]. Each derivation gives different insights. Before proceeding we define the basic quantities and present our formula; further details are in [14].

Consider a right handed coordinate system (Z, X, Y) as shown for a bunch compressor in Fig. 1. The relevant fields are $\mathcal{E} := (E_Z, E_X, B)$ where B is the Y component of the magnetic field. We assume a fixed Y distribution, $H(Y)$, thus the basic equation for $\mathcal{E} = \mathcal{E}(\mathbf{R}, Y, u)$, where $\mathbf{R} := (Z, X)$ and $u := ct$, is

$$(\partial_Z^2 + \partial_X^2 - \partial_u^2)\mathcal{E} = H(Y)\mathcal{S}(\mathbf{R}, u) \tag{1}$$

with shielding boundary condition

$$\mathcal{E}(\mathbf{R}, Y = \pm g, u) = 0. \tag{2}$$

Here

$$\mathcal{S} = (\nabla \rho_L/\epsilon_0 + \mu_0 c \partial \mathbf{J}_L/\partial u, -\mu_0(\nabla \times \mathbf{J}_L)_Y) \tag{3}$$

where $\rho_L(\mathbf{R}, u)$ is the 2D lab frame spatial density, $\mathbf{J}_L(\mathbf{R}, u) = \int \mathbf{V} f_L(\mathbf{R}, \mathbf{V}, u) d\mathbf{V}$ is the current density per unit charge and $f_L(\mathbf{R}, \mathbf{V}, u)$ is the lab frame phase space density.
Our basic formula for $\mathcal{E}$, averaged over Y, is

$$\mathcal{E}(\mathbf{R}, u) = -\frac{Q}{2\pi} \sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta \; \mathcal{S}(\hat{\mathbf{R}}, v, k), \tag{4}$$

where $a_k = (-1)^k(1 - \delta_{k0}/2)$, $\hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u-v)^2 - (kh)^2}(\cos\theta, \sin\theta)$, $h = 2g$, and Q is the total charge. The field calculation is the most expensive part of our calculations and even though our formula is simple, its implementation has some subtleties.

First of all the field calculations need to be done very accurately. This is because the fields are large but the CSR terms in the equations of motion, to be discussed below, are small. We have made progress on this problem by calculating the CSR terms directly.

Second, the charge density and associated derivatives need to be calculated accurately. However in a phase space approach it is difficult to determine the support of the phase space density as previously mentioned. As a result we have found it important to study a Monte Carlo approach before going to a phase space approach, as the charge density can be reasonably calculated with particles. To evolve a collection of particles, it is clear we need to calculate the CSR terms at the particle positions. Bassi has found it is much too expensive to do this at each particle position, and so a moving grid and a biquadratic interpolation have been implemented. In addition, the moving grid and interpolation are necessary for the Vlasov-Maxwell calculation.

A2. Lab to Beam Transformation

Our basic formula for the fields is in the lab frame, but we currently believe it is best to do the particle evolution in a beam frame interaction picture. The lab to beam transformation is

$$\mathbf{R} = \bar{\mathbf{R}}(s) + x\mathbf{n}(s), \tag{5}$$

where

$$\bar{\mathbf{R}}(s) = (\bar{Z}(s), \bar{X}(s)), \quad \mathbf{t}(s) = \bar{\mathbf{R}}'(s), \quad \mathbf{n}(s) = (-\bar{X}'(s), \bar{Z}'(s)), \quad \kappa(s) = -\alpha B_e(\bar{Z}(s)). \tag{6}$$

Here $\bar{\mathbf{R}}(s)$ is the design orbit, $\mathbf{t}$ and $\mathbf{n}$ are unit vectors shown in Fig 1, s is the distance along the design orbit, $B_e(Z)$ is the magnetic field of the dipoles and $\alpha =\mid e \mid /m\bar{\gamma}\bar{v}$. We have done a careful study of the transformation between lab and beam frame phase space densities which leads to the following approximate relations

$$\begin{aligned} \rho_L(\mathbf{R}, u) &= \rho(\mathbf{r}, \beta u) \, , \\ \mathbf{J}_L(\mathbf{R}, u) &= \beta c[\rho(\mathbf{r}, \beta u)\mathbf{t}(\beta u + z) + \tau(\mathbf{r}, \beta u)\mathbf{n}(\beta u + z)], \end{aligned} \tag{7}$$

where

$$\rho(\mathbf{r}, s) = \int d\mathbf{p} f(\mathbf{r}, \mathbf{p}, s) \qquad\qquad \tau(\mathbf{r}, s) = \int d\mathbf{p} p_x f(\mathbf{r}, \mathbf{p}, s) \tag{8}$$

and $f(\mathbf{r},\mathbf{p},s)$ is the beam frame phase space density. Here $\bar{v}=\beta c$ is the speed on the design orbit, $\mathbf{r}=(z,x)$ where $z(s)=s-\bar{v}t(s)$ and $\mathbf{p}=(p_z,p_x)$ where $p_z=(E-\bar{E})/\bar{E}$ and $p_x=v_x/\bar{v}$ and $t(s)$ is the arrival time at position s, not to be confused with the unit tangent vector $\mathbf{t}(s)$. We have chosen to study the beam evolution in the beam frame because the phase space variables remain small and to good approximation the beam frame equations are linear without CSR. The equations of motion can be written:

$$\begin{aligned} z' &= -\kappa(s)x, & p_z' &= F_z, \\ x' &= p_x, & p_x' &= \kappa(s)p_z + F_{ex} + F_x, \end{aligned} \tag{9}$$

where

$$\begin{aligned} F_z &= \frac{e}{\bar{v}\bar{E}}\mathbf{V}\cdot\mathbf{E}, \\ F_{ex} &= -\kappa^2(s)x + \alpha[B_e(\bar{Z}(s) - x\bar{X}'(s)) - B_e(\bar{Z}(s))] \\ F_x &= \frac{e}{\bar{E}\bar{\beta}^2}\Big[-\bar{X}'(s)E_Z + \bar{Z}'(s)E_X + \bar{v}B - \frac{\bar{\beta}p_x}{c}\mathbf{V}\cdot\mathbf{E}\Big]. \end{aligned}$$

Here $\mathbf{V}=\bar{v}(\mathbf{t}(s)+p_x\mathbf{n}(s))$ and $\mathbf{E}(\mathbf{R},u)=(E_Z,E_X)$ and $B(\mathbf{R},u)$ are evaluated at $\mathbf{R}=\bar{\mathbf{R}}(s)+x\mathbf{n}(s)$ and $u=(s-z)/\bar{\beta}$. The CSR terms mentioned above are now seen to be F_z and F_x, where the former is basically the power and the latter is basically the transverse force.

We have studied the F_{ex} term and found it to be small enough to ignore. A more careful analysis of this term is in progress. Due to the presence of strong cancellation effects at different levels in F_x we are now carefully studying the most efficient way to calculate it on a grid.

The principle solution matrix (PSM) ($\Phi(s|s_0)$, $\Phi(s_0|s_0)=I$) for (9) without CSR is easily written in terms of the lattice functions, that is the dispersion function, $D(s)$, and $R_{56}(s)$ both of which are simply integrals of the curvature κ. The interaction picture is defined in terms of the PSM by the transformation $\zeta\rightarrow\zeta_0$ via

$$\zeta = \Phi(s|0)\zeta_0, \tag{10}$$

where $\zeta=(z,p_z,x,p_x)$ and similarly for ζ_0. The equations of motion in the interaction picture become:

$$\begin{aligned} z_0' &= -R_{56}(s)F_z - D(s)F_x, & p_{z0}' &= F_z, \\ x_0' &= (sD'(s)-D(s))F_z - sF_x, & p_{x0}' &= -D'(s)F_z + F_x, \end{aligned} \tag{11}$$

A draft manuscript has been written on these matters [18] and can be found at our ILC web site.

A3. Liouville-Maxwell Model

As an important step toward the full Vlasov-Maxwell treatment, we have done an extensive study of what we call the Liouville-Maxwell Approximation (LMA). This was proposed in IIIB above. In the LMA, we take the fields to be those created by the unperturbed bunch and then study the evolution of a particle bunch is this field, that is, we study the solution of the beam frame Liouville equation. It is clear that the LMA will be a good approximation to the self-consistent case if the CSR is small enough. Futhermore, we believe that the parameters for the bunch compressor that we are studying are such that it will be a rough approximation. Since the above PSM is easily computed upfront, we found that it is computationally advantageous to calculate the beam evolution in the beam frame interaction picture where the coordinates evolve slowly.

Up to this point all our LMA calculations have been done using a Monte Carlo approach. Millions of particles can be followed through the bunch compressor with a modest amount of computer time and this gives enough data to accurately construct the particle density as well as low order moments of the phase space density. Our next step in this context will be to integrate the Liouville equation using the PF method we have developed to calculate the phase space density [9].

Our first approach ignores the beam frame transverse force F_x. We call this model LMA1. It is of intrinsic value as well as being important as a baseline for the study of transverse effects. Here we have detailed calculations of the mean energy loss and its variance as well as the transverse emittance. Bassi now has a very fast code for this calculation and our results are in reasonable agreement with the Zeuthen benchmark results [19] (reasonable agreement is all that should be expected as the benchmark results have significant variation and furthermore our calculations are not yet self-consistent). Our preliminary results were presented at PAC2005 and FEL2005. During the preparation for PAC we discovered the importance of calculating F_z on a moving grid and using interpolation for off-grid points; a technique we also use for F_x. A paper [20] is in preparation showing these results.

Accurate calculation of the transverse force, F_x, is delicate because both the electric and magnetic field contributions to the transverse force are large, whereas the transverse force is relatively small. Here we have found that this issue is best treated at the source level where the subtraction of large terms is more easily handled. Bassi has completed a careful study of the transverse force and has found that the effect is indeed small [20]. It does have a significant effect on the transverse emittance and in a positive way in that it is 25% smaller than in LMA1. The calculations of R. Li and A. Kabel also show a decrease. In addition, we have found that proper attention to the magnet edge effects is quite important, i.e., the F_{ex} term in (9) can be ignored.

In summary, we now have a very fast code for the Monte Carlo LMA calculation which includes the transverse force. An appropriate moving grid has been chosen reflecting the support of the charge density. The central feature of our calculation is the upfront calculation of F_z and F_x on the grid. Calculations off the grid are done using a biquadratic interpolation. It is now quite fast to follow millions of particles through the bunch compressor. A variance reduction technique then gives very accurate statistics for mean energy loss and emittance. We have gained important information on how the charge density evolves which will be helpful for the self-consistent calculation. A paper [20] is in preparation.

It is important for the Vlasov-Maxwell calculation to understand the support of the PSD, a nontrivial task due the strong correlation between the phase space variables. We are now studying this in the LMA.

A4. Vlasov-Maxwell Calculation

Based on our experience with the field calculation in the LMA we are now confident we can self-consistently calculate the CSR using particles (Monte Carlo). Here the most important new piece needed is an accurate calculation of the charge density and its derivatives from the particle positions at predetermined times. In order to determine the number of particles needed to accurately build the charge density, we have integrated the equations of motion without CSR and then used the particle positions to compute a charge density by depositing the particles on the grid from the LMA calculation above. This is then interpolated to obtain the source, which includes derivatives of the density, at off grid points. The field calculation can now be done as in the LMA and the two compared. We have found that with roughly 10 million particles, the field calculation in LMA is accurately reproduced.

B. Damping Ring Instabilities

Considerable progress was made on items 1, 2 and 4 of Section V of the proposed work.

Marc Salas wrote a very nice Master's thesis on items 1 and 2 ([21]). A master's thesis in our department is supposed to be equivalent to 2 courses (6 units). Salas put in much more effort than this. The algorithm developed in [9] and the associated code written by Warnock was modified for the calculations. A subroutine was written by Warnock to compute the Haïssinski equilibrium distribution incorporating both machine wake and CSR and this was used as a starting point for most of the calculations. Salas studied two machines; the SLAC damping ring and the VUV ring at Brookhaven. In both cases he studied the interaction between CSR and a machine wake. In the case of the SLAC damping ring, Salas reproduced the sawtooth instability of [9]. He then added the CSR and found it has a small effect on the onset of instability of the Haïssinski equilibrium and a slightly bigger effect on the onset of the sawtooth instability. For the VUV ring Salas first studied the CSR alone in the spirit of [10]. Here he found the beautiful bursting modes at high current. However, for realistic VUV parameters he found that the CSR had only a small effect. Warnock, Bassi, Heinemann and I provided much support and we all learned alot, but more work is needed. This will be discussed below.

With regard to item 4, Heinemann made significant progress, [22], in understanding the results of the first Saldin, et al., paper, [11]. The problem of the singularity and the renormalization does not arise in his approach which uses the advanced as well as the retarded potentials. Heinemann writes in the introduction to [22]:
"We here reconsider the model from [11, 12] of a beam of electrons which move with the same constant speed and on the same curve. The important achievement of Saldin et al. is the estimation of the radiative work (RW) of the beam and in particular the contribution from coherent synchrotron radiation (CSR), i.e. the non-Larmor contribution. Saldin et al. accomplish this by using the retarded Lienard-Wiechert field and the following three items: (i) the *small-angle approximation* by which one assumes that the angle (hence the distance) between electrons is small; (ii) the *high-energy approximation* by which one assumes that the electrons have high energy; (iii) the subtraction, i.e. renormalization, of the so called velocity term which goes to infinity in the limit of zero distance of the electrons. The approximations (i) and (ii) are granted by the fact that in the context of CSR one is particularly interested in *short* bunched beams of *high* energy. Note also that Saldin et al. study two different perturbation theories, i.e. two alternative versions of items (i) and (ii): in one version the distance is very small and in the other version it is bigger but still small (for more details, see the main text)."

"The motivation for the present paper was to check the renormalization procedure and the estimated RW in [11]. In addition we will show that the renormalization procedure can be avoided if one brings the *advanced* Lienard-Wiechert field into play. In fact we will define the RW differently from [11]: while Saldin et al. base their study on the *retarded* Lienard-

Wiechert field, we will base ours on half the difference between the retarded and advanced Lienard-Wiechert fields (one often calls this the *radiation field* and so do we). The main virtue of the radiation field, discovered in the 1930's, is that it takes into account exactly that part of the work which is due to radiation, i.e. is irreversible. A great merit of the radiation field is that it allows to compute the RW nonperturbatively which in turn makes checking the perturbation theories of Saldin et al. especially easy. Note that the renormalization procedure of Saldin et al. is modelled after the linearly accelerated motion (actually they call it a 'trick') and so they do not pursue results which hold beyond perturbation theory (accordingly, in the nonperturbative regime Saldin et al. get results different from ours)."

"The connection between the method of Saldin et al. and our method can be further characterized as follows (see also [23, 24]). Saldin et al base their calculation of the RW on the work done by the retarded Lienard-Wiechert field and this work has a radiative part and a nonradiative part (thus from this standpoint it is apriori clear that something has to be renormalized). The radiative part comes from the radiation field, i.e. half the difference between the retarded and advanced fields and it is irreversible. Note that the radiation field is *finite* even at the position of the electrons (the discovery of this fact is usually attributed to Wentzel [25] and it came to full prominence with Dirac [26]). The nonradiative part (which Saldin et al. rightfully call *nondissipative*) comes from half the sum of the retarded and advanced fields which feeds energy into the electromagnetic field in a reversible way because it changes sign under time reversal (note also that, in classical electron theory, the nonradiative part is linked with the concept of the *electromagnetic mass* of the electron). Since the Saldin-Schneidmiller-Yurkov model is one-dimensional in the sense that the beam is concentrated on the above mentioned curve the nonradiative part of the work is infinite (this is similar to the fact that the electric field of a line charge is infinite inside the line). Thus Saldin et al. are urged to renormalize it and so, roughly speaking, while *they* add a renormalization term, *we* have the advanced field which does the job."

"Our paper is structured as follows. We first define the Saldin-Schneidmiller-Yurkov model and in particular the RW. Then we study the RW nonperturbatively. The remaining parts of the paper deal with perturbation theory, in particular we compare with the perturbative results of Saldin et al. Of course, in contrast to [11, Section 2], no renormalization is to be done in our paper, i.e. item (iii) does not arise in our calculations. Since our paper also serves pedagogical purposes we cover the following three issues in some detail: firstly we carefully study in the Appendix the retarded and advanced times and the retarded and advanced angles; secondly we study *all four* contributions to the RW (the two relativistic Larmor terms and the two CSR terms) and, thirdly, we do a small–angle approximation which is similar but different from the ones in [11, Section 2]."

Next year Project Activities and Deliverables

Our main focus for the next year is the creation of two self-consistent codes for the bunch compressor, one based on Monte Carlo (MC) and one based on the PF method to compute the phase space density (PSD). This will fullfill the proposed work of IIIB. The main road block to the 4D PSD calculation is the creation of a moving grid that captures its main support. Our work up to this point indicates this is a tough problem. We will proceed simultaneously on the MC and PF codes. Once the codes are in place, we will be able to nail down the issue of transverse emittance growth in the Zeuthen 5GeV benchmark bunch compressor and then begin to explore other bunch compressor parameters such as the Zeuthen 500 MeV case and potential ILC parameters.

The main issues for the MC code are (1) the computation of a smooth spatial density from the particle positions and its 3D interpolation in space and time and (2) the creation of a parallel code once the serial code is in good shape. There is much current literature on the computation of densities from data in both the statistics and numerical analysis communities; literature we are now exploring, e.g. [27]. We have experts here both in statistics and numerical analysis who will be able to help us. In addition, we have access to parallel machines and experts on parallel computing.

The main issues for the PF method are the support problem for the PSD mentioned above and the creation of a 4D interpolation for its evaluation, which is key to the PF method of local characteristics. The MC calculations will give considerable information about the evolution of the support of the PSD and we hope that this will lead to an efficient method for the construction of the 4D moving grid. We will likely start by calulating the PSD in the LMA model where the field calculation can be done up front.

Interpolation techniques in 2, 3 and 4 dimensions are important to us and a significant effort will be devoted to the study of modern state-of-the-art algorithms. For example, there is much current interest in radial basis functions, [28, 29, 30], and Bob Warnock and Marco Venturini (LBL) have begun an investigation of their use in CSR problems. Of course splines continue to play an important role in modern numerical analysis, [31, 32].

As mentioned in the progress section, Heinemann made substantial progress in understanding the results of [11] by using the retarded-advanced field introduced by Dirac and Schwinger. As a final step he will rederive the formula Borland [4] uses and see if his calculation gives any improvement. He will then study the wiggler paper, [12], and proceed as outlined in item V.4 of the proposal. The results of [11, 12], are used in several CSR codes, e.g., [4]. Their contribution is important because it leads to fast CSR codes in a 2D phase space. However, in the end, it is not clear how reliable these 2D models are. Once our self-consistent codes above are ready, we will be able to investigate the accuracy of these codes. In addition we will investigate a 2D approximation starting from our basic formula (4).

The work of Salas on damping ring instabilities was discussed in the progress section. While he did a very nice piece of work, it is only a start. Interesting future developments of Salas' work are: a deeper study of the microstructures responsible for the emission of bursts of radiation and an improvement of the algorithm used (more accurate interpolation schemes, splitting methods and finite different schemes.) The code is now in place for the study of damping ring instabilities with both machine wakes and CSR, from both bending magnet and wigglers, and could now be applied to ILC damping ring parameters.

Subsequent year Project Activities and Deliverables

The project activities above are ambitious and will likely spill over into a second year. Other issues that we plan to address are:

- We have several ideas to improve the field calculations. These are too technical to discuss here. Some of these may be done as we create the two self-consistent codes. But likely that will occur in a refinement stage once the basic codes are in place.
- Verification and validation of codes and benchmarking is very important as is discussed in the Working Group 1 report from the Snowmass workshop on the ILC and as discussed in a recent Physics Today article [33]. Once our codes are in reasonable shape we will benchmark them with respect to other CSR codes [15].

- It is not clear at this point how robust our code will be in terms of applying it to other bunch compressors and to changes in the initial distribution in the bunch compressor we are currently studying. Once our self-consistent codes are in reasonable shape we will begin to investigate other set of parameters.

Budget justification:

Warnock is not asking for financial support; he finds it sufficient to get theoretical and numerical collaboration from Ellison, Bassi and Heinemann. Ellison is not asking for financial support either as CSR is one item in the research of his current DOE grant - DE-FG03-99ER41104 for "Investigations of Beam Dynamics Issues at Current and Future Accelerators". In addition, Ellison has funds in his DOE grant to partially support the CSR work of PostDoc Bassi.

We are requesting \$52.4K for next year and \$41.4K for the subsequent year to fill out the support of Bassi's CSR work and to partially support Heinemann as a graduate research assistant. Because of reasons known to my DOE program manager, Phil Debenham, Bassi will not have funding for approximately 2 months next year. This is the main reason for the higher funding request for next year. Fringe benefits for a PostDoc are charged at 18.5% on salary and for a research assistant at 1% on salary plus health insurance at \$1223/year. In addition a research assistant salary is required by UNM to include \$3764 for tuition costs. UNM's indirect cost rate is 50%. I have scaled the subsequent year by 5%.

Two-year budget, in then-year K$

Institution: Institution 1

Item	Next year	Subsequent year	Total
Other Professionals	7000	0	7000
Graduate Students	24164	25130	49294
Undergraduate Students	0	0	0
Total Salaries and Wages	31164	25130	56294
Fringe Benefits	2760	1475	4235
Total Salaries, Wages and Fringe Benefits	33924	26605	60529
Equipment	0	0	0
Travel	1000	1000	2000
Materials and Supplies	0	0	0
Other direct costs	0	0	0
Total direct costs	34924	27605	62529
Indirect costs at 50%	17462	13803	31265
Total direct and indirect costs	52386	41408	93794

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