Project name

Investigation of Novel Schemes for Injection/Extraction Kickers (LCRD 2.22)

Classification (accelerator/detector:subsystem)

Accelerator

Institution(s) and personnel

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Project Overview

The 2820 bunches of an ILC pulse would require an unacceptably large damping ring if the 337 ns linac bunch spacing were used in the damping ring. As a result, the TESLA 500 GeV design1 called for 20 ns bunch separation in a 17 km circumference damping ring; a fast kicker would deflect individual bunches on injection or extraction, leaving the orbits of adjacent bunches in the damping ring undisturbed. A number of the kicker designs which had been considered involve the creation of individual magnetic field pulses of sufficiently short duration so that only one bunch is influenced by a pulse. The demands of short rise/fall times and pulse-to-pulse stability are challenging; a system capable of generating shorter pulses would allow the construction of a smaller damping ring.

It is interesting to consider a design in which the pulsed kicker is replaced by a low-$Q$ RF device filled with a broadband signal whose amplitudes, frequencies, and phases correspond to the Fourier components of a periodic, narrow pulse. Instead of energizing the system only when a bunch was about to be injected (or extracted) to the damping ring, the device would run continuously. This might allow the frequencies, phases, and relative amplitudes of the impulse to be determined with great precision. With a properly

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chosen set of parameters, the system would kick every $M^{th}$ bunch in a train, leaving undisturbed the train’s other $(M-1)$ bunches. Injection (or extraction) of an entire bunch train would be completed by the end of the $M^{th}$ orbit through the system. We are studying a kicker with $M = 60$, admitting the construction of a 6 km circumference damping ring.

We will assume that the damping ring beam is organized as a number of bunch trains with an inter-train gap between the tail of one bunch train and the head of the next. The kicker system would be installed in a bypass section of ring. During injection, a deflector system would route the beam through the bypass. Once injection was completed, each deflector would be turned off during the passage of a gap between bunch trains. The beam would then orbit in the damping ring, bypassing the kicker. At extraction, the deflectors would be energized again, routing the beam through the kicker for extraction.

**Brief description of the Fourier series pulse compression kicker**

A general discussion of a Fourier-series kicker system can be found on the web. We will focus on a particular implementation in which an RF amplifier sends a broadband signal down a waveguide to a $Q \approx 25$ RF structure. Dispersion in the waveguide shifts the relative phases of the Fourier components of the signal so that it is compressed: RF power arrives at the RF structure in short, periodic bursts, filling it in order to eject the target bunch without disturbing adjacent bunches. The RF structure is able to store energy, so its maximum field strength is approximately 20 times greater than the maximum field strength in the downstream end of the waveguide.

A schematic representation of the kicker is shown in Figure 1. Our studies assume the values for the kicker’s parameters shown in Table 1.

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Figure 1: Schematic representation of a Fourier series pulse compression kicker.

Table 1: Fourier series pulse compression kicker parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main linac bunch frequency</td>
<td>$f_L$ ($\omega_L \equiv 2\pi f_L$)</td>
<td>3 MHz</td>
</tr>
<tr>
<td>Damping ring bunch frequency</td>
<td>$f_{DR}$ ($\omega_{DR} \equiv 2\pi f_{DR}$)</td>
<td>180 MHz</td>
</tr>
<tr>
<td>RF structure center frequency</td>
<td>$f_{RF}$ ($\omega_{RF} \equiv 2\pi f_{RF}$)</td>
<td>1845 MHz</td>
</tr>
<tr>
<td>RF structure $Q$</td>
<td>$Q$</td>
<td>25</td>
</tr>
<tr>
<td>Waveguide cutoff frequency</td>
<td>$f_{cutoff}$</td>
<td>1300 MHz</td>
</tr>
<tr>
<td>On field integral</td>
<td>$A(0)$ ($100 \pm .07$) Gauss-meters</td>
<td></td>
</tr>
<tr>
<td>Off field integral</td>
<td>$A(t)$ ($0 \pm .07$) Gauss-meters</td>
<td></td>
</tr>
<tr>
<td>$f_{DR}/f_L$</td>
<td>$N$</td>
<td>60</td>
</tr>
<tr>
<td>$f_{RF}/f_{DR}$</td>
<td>$\Gamma$</td>
<td>10.25</td>
</tr>
<tr>
<td>$f_{RF}/f_L$</td>
<td>$\Gamma N$</td>
<td>615</td>
</tr>
<tr>
<td>Bunch length</td>
<td>$\delta_B$ or $\tau_B$</td>
<td>$\pm 6 \text{ mm } - \pm 20 \text{ ps}$</td>
</tr>
</tbody>
</table>

Kicker impulse function

We have been studying an impulse function of the following form:
The initial term, a normalized ratio of squares of sine functions, is shown in Figure 2 for an interval ±25 ns around the kicking peak. This envelope function sets the spacing between kicking peaks and major zeroes. The parameter \( N \) is the ratio of the bunch spacing in the main linac and damping ring.

Besides controlling the locations of the major zeroes, this envelope function also flattens the kicker’s field integral \( A(t) \) in the vicinity of the zeroes since the ratio of the squares of sine functions also has zero slope at major zeroes in \( A(t) \). This way, the variation in field integral between the center and ends of an individual (unkicked) bunch is insignificant.

**Frequency spectrum of envelope function**

The frequency spectrum of the envelope function is shown in Figure 3. Because the kicker signal is periodic, the envelope function’s spectrum is discrete, with individual Fourier components separated by the main linac bunch frequency \( f_L \), 3 MHz in the present case. Note that the spectrum peaks at zero frequency, then descends linearly, becoming exactly zero at \( Nf_L = 180 \) MHz.

\[
A(t) = \frac{1}{N^2} \frac{\sin^2 \left( \frac{1}{2} \omega_{Df} t \right)}{\sin^2 \left( \frac{1}{2} \omega_f t \right)} \cos \left( \omega_{Rf} t \right) = \frac{1}{N^2} \frac{\sin^2 \left( \frac{1}{2} N \omega_L t \right)}{\sin^2 \left( \frac{1}{2} \omega_L t \right)} \cos \left( \Gamma N \omega_L t \right). \tag{1}
\]

Figure 2: Envelope function ±25 ns around the kicking peak. Unkicked bunches pass through the kicker during zeroes in the field integral.
Fourier coefficients of
\[ A = \frac{1}{N^2} \frac{\sin^2\left(\frac{1}{2} N \omega_c t\right)}{\sin^2\left(\frac{1}{2} \omega_z t\right)} \]

Figure 3: Fourier amplitudes of the envelope function; plot extends from 0 to 300 MHz. Note that there are 60 non-zero amplitudes and that the spacing between frequency components is \( f_L = 3 \) MHz.
Because $A(t)$ is an even function, its Fourier expansion consists entirely of cosines.

**High frequency modulating function**

We must choose the frequency of the modulating cosine so that $\cos(\Gamma N \omega L t) = +1$ whenever a bunch is to be kicked, of course. But we can further flatten the field integral $A(t)$ at the major zeroes immediately before and after the kicking pick with an appropriate choice of $f_{RF}$. To do this we select a cavity center frequency $f_{RF}$ that makes the modulating cosine zero as the first (or last) unkedded bunch traverses the kicker, when $t = \pm 1 / f_{DR}$. With a value of $f_{RF}$ satisfying this requirement, we will have the modulating cosine run through the values of 0, -1, 0, +1, … at successive major zeroes. Note that the value of $\Gamma$ must differ from an integer by 0.25 in order for the modulating cosine to be zero at the first major zero. In addition, $N$ must be an integer multiple of 4 in order for the modulating cosine to assume the value +1 at all kicking peaks.

The resulting field integral (using $f_{RF} = 1845$ MHz) is shown in Figure 4 for an interval $\pm 25$ ns around the kicking peak.

![Figure 4: Kicker field integral $A(t)$ in an interval $\pm 25$ ns around the kicking peak.](image)

**Frequency spectrum of the impulse, including effects of the modulating function**

The useful trigonometric identity $\cos(a) \cos(b) = \cos(a+b)/2 + \cos(a-b)/2$ lets us immediately write the Fourier expansion of $A(t)$ in terms of the Fourier series for the
envelope function described above. A graph of the Fourier amplitudes of $A(t)$ is shown in Figure 5. These amplitudes will yield a unit strength kicking pulse.

![Fourier coefficients of $A(t)$](image)

**Figure 5:** Fourier amplitudes of the field integral impulse $A(t)$. The peak is at frequency $f_{RF} = 1845$ MHz; the amplitudes become identically zero at $f_{RF} \pm 180$ MHz. These amplitudes will yield a unit strength kicking pulse.

### Details of the kicking peak

Figure 6 shows the kicking peak for the time interval (-50 ps, 50 ps). The shape is dominated by the high frequency modulation term $\cos(\Gamma N_0 \omega_L t)$. The kick drops rapidly away from its maximum at $t = 0$. This raises two separate issues. Is the kicker’s injection efficiency adequate? Are extracted bunches sufficiently free of kicker-induced distortion so that head-center-tail effects do not degrade the ILC’s luminosity? Recall that the RMS width of a damping ring bunch is $\pm 20$ ps.

One promising strategy would be to install a corrector in the injection and extraction lines in order to compensate for the time dependence of the impulse delivered to a bunch. A single-frequency RF system running at $f_{RF}$ could deliver an impulse of the opposite sign to bunches shortly before/after injection/extraction. The residual error in kick after a corrector, shown in Figure 7, is about a third as large as the required kicker precision of 0.07%.
Figure 6: Kicking peak in a time interval ± 50 ps around \( t = 0 \). RMS bunch length is 20 ps.

Figure 7: Fractional kick error, including an injection/extraction line corrector, in a time interval ± 50 ps around \( t = 0 \). Maximum allowable error is 0.0007, roughly three times the error at 50 ps.
Modifications to the kicker RF system intended to flatten the kicking peak will generally increase the required amplifier bandwidth, through introduction of a low frequency tail, higher frequency bands in the vicinity of harmonics of the cavity center frequency, or both.

**Details of the major zeroes**

Figures 8a, 8b, 8c, and 8d show the impulse function in ±50 ps intervals around the first four major zeroes. Note that the choice of modulation frequency makes the error in the vicinity of the zeroes cycle through four different shapes: descending cubic, downwards quadratic, ascending cubic, and upwards quadratic. This may have the virtue of canceling some of the accumulated errors for bunches which make multiple passes through the kicker before being extracted. As it stands, the kick error when a bunch transits the kicker during a single orbit is small compared to the maximum allowable error of 0.0007. Details of this will depend on the tune of the damping ring.

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**Figure 8:** Impulse function in a time interval ± 50 ps around the first four major zeroes. Note that the maximum allowable deviation from zero is 0.0007, substantially larger than the simulated kicker error.
Low-$Q$ cavity

The Fourier series kicker needs to apply a ±10% bandwidth impulse to kicked bunches. It is an interesting technical challenge to devise a method to sum the effects of the various frequency components. An early (naïve!) conception of the kicker\(^3\) used one cavity for each frequency component, letting the beam sum the effects of the individual Fourier components. Another idea\(^4\), originally described by Joe Rogers, uses a low-$Q$ cavity that can support the range of frequencies comprising the kicking pulse.

Amplitude and phase lag of cavity response to driving signal

We have not modeled the required cavity in any detail. In our simulations we assume it has $Q = 25$, is linear, and produces fields of adequate quality. We ignore the effects of reflections at its input port and describe the cavity’s response to a driving signal of frequency $\omega$ and amplitude $S_{drive}$ this way:

$$
A(t) = A_{cavity} \cos(\omega t - \phi) = \frac{S_{drive}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{2\omega_0\omega}{Q}\right)^2}} \cos(\omega t - \phi). 
$$

(2)

Here $\omega_0$ is the cavity’s center frequency; the phase lag $\phi$ is defined as

$$
\phi = \tan^{-1}\left(\frac{2\omega_0\omega}{Q(\omega_0^2 - \omega^2)}\right).
$$

(3)

The ratio $A(t)/S_{drive}$ over the required bandwidth is shown in Figure 9. The phase lag $\phi$ is shown in Figure 10. (These are the equations describing the response of a damped, driven harmonic oscillator.)

Fourier amplitudes and phases of driving signal arriving at the cavity

Since the cavity stores energy and its response is frequency dependent, dividing the Fourier amplitudes shown in Figure 5 by the response function $A(t)/S_{drive}$ generates the distribution of Fourier amplitudes that must be delivered by the amplifier to the downstream end of the waveguide to generate a unit-strength kicking pulse. This spectrum is shown in Figure 11. (Bear in mind that we have not yet modeled the frequency-dependent transfer efficiency of power from the waveguide into the cavity.)

The phase relationship between input and response changes considerably as the frequency crosses through resonance. Including the frequency-dependent phase of the

\(^3\) Ibid.

cavity response allows us to plot the required field as a function of time at the downstream end of the waveguide. This is shown in Figure 12.

Figure 9: Cavity response to driving fields, assuming a $Q = 25$ cavity with center frequency 1845 MHz.

Figure 10: Phase lag between cavity response and driving fields.
Figure 11: Fourier amplitudes required at the downstream end of the waveguide to generate a unit-strength kicking peak in the RF cavity. The amplitudes were derived by dividing the coefficients in Figure 5 by the cavity’s frequency-dependent response.

Figure 12: Field at the downstream end of the waveguide that will generate a unit-strength kicking peak in the RF cavity. Note the difference in the maximum amplitude in the waveguide in comparison with the cavity field, shown in Figure 4.
It is possible that the kicker system’s sensitivity to errors might be reduced (at the expense of greater input power) if the cavity center frequency were chosen to be above or below the kicking pulse’s frequency band. This is worth investigating in detail at a later time.

Waveguide

We consider a rectangular waveguide with cutoff frequency $f_{\text{cutoff}} = 1300$ MHz driven in its dominant mode\(^5\). The cutoff frequency depends on the waveguide’s geometry, shown schematically in Figure 13. When $b > a$, we have $f_{\text{cutoff}} = c/(2b)$, where $c$ is the speed of light in vacuum. Our 1300 MHz waveguide has $b \approx 11.5$ cm.

![Waveguide geometry](image)

Figure 13: Waveguide geometry. Power propagates in the $+z$ direction. For this waveguide, $f_{\text{cutoff}} = c/(2b) = 1300$ MHz.

Group velocity

Besides providing a path between the amplifier and the RF structure, the waveguide serves as the dispersive transport that compresses the amplifier signal. The waveguide’s group velocity as a function of frequency is

$$v_g = c\sqrt{1 - \left(\frac{f_{\text{cutoff}}}{f}\right)^2} = c\sqrt{1 - \left(\frac{\omega_{\text{cutoff}}}{\omega}\right)^2}. \quad (4)$$

Figures 14 and 15 show plots of $v_g(f)$ in the frequency ranges 0 to 4000 MHz and 1665 to 2025 MHz.

Energy pumped into the upstream end of the waveguide will propagate down the guide with speed equal to the group velocity. Because $v_g(f)$ is frequency dependent, the relative phases of various Fourier components will vary with position along the length of the waveguide in a way that allows the delivery of a compressed pulse to the cavity.
**Phase lag as a function of frequency and position**

Imagine that a single Fourier component with angular frequency $\omega$ is propagating in the $z$ direction. We can write its amplitude at a fixed $z$ position as $a \cos[\omega(t - t_{\text{propagation}})]$. The time delay $t_{\text{propagation}}$ because power flows down the waveguide at the (finite) group velocity: we expect $t_{\text{propagation}} = z/v_g$.

Since the group velocity is frequency dependent, the propagation times for different Fourier components flowing through a waveguide will vary so that their relative phases at the end of the waveguide will be different from their initial values at the entrance to the waveguide. We have

$$t_{\text{propagation}}(z, \omega) = \frac{z}{v_g(\omega)} = \frac{z}{c} \left[ 1 - \left( \frac{\omega_{\text{cutoff}}}{\omega} \right)^2 \right]^{1/2}. \quad (5)$$

The phase lag for a Fourier component of angular frequency $\omega$ relative to one of infinite frequency is, as a function of distance $z$ along the waveguide,

$$\Delta \varphi(z, \omega) = \omega \left[ t_{\text{propagation}}(z, \omega) - t_{\text{propagation}}(z, \infty) \right] = \omega z \left[ \frac{1}{v_g(\omega)} - \frac{1}{c} \right]. \quad (6)$$

The relative change in phase per meter of waveguide for adjacent Fourier components (separated in frequency by 3 MHz) is plotted in Figure 16. From the plot one can see, for example, that increasing the waveguide length by one meter will increase the phase lag of the 1800 MHz Fourier component by $\sim 4^\circ$ relative to that of the adjacent (1803 MHz) component.

Shown in Figure 17a are plots of the field amplitudes in the waveguide as functions of time at $z$ positions increased in 5 meter intervals. We have adjusted the graphs’ origins to keep the amplitude distributions near the center of each plot. The plots are presented in reverse time-order, with the compressed pulse at the downstream end of the waveguide appearing first. Figure 17b shows similar plots, but spaced at 20 meter intervals. Figure 17c is an enlarged version of the 80 meter plot from Figure 17b. Unusual features in the graphs are artifacts.
Notice that the peak power required from an amplifier at the upstream end of an 80-meter waveguide is not particularly different from the amplifier’s average power output. The field amplitude at the entrance to the waveguide is roughly one percent as large as the maximum field inside the RF kicking structure.

We have not yet optimized the length or cutoff frequency of the waveguide. Operating with a cutoff frequency closer to the cavity’s center frequency would provide more dispersion per meter, allowing use of a shorter waveguide at the expense of greater sensitivity to inaccuracies in waveguide geometry.
Figure 17a: Fields at different distances from the downstream end of the waveguide, shown as functions of time.
Figure 17b: Fields at different distances from the downstream end of the waveguide, shown as functions of time.
Figure 17c: Field 80 meters from the downstream end of the waveguide, shown as a function of time.
Power amplifier

We will assume that a 100 Gauss-meter impulse (3 MeV/c) is adequate. The kicking impulse under consideration requires an amplifier capable of generating signals in the frequency range \((1800 \pm 180)\) MHz.

Estimate of required amplifier power

Recall that the electric and magnetic fields in a resonant structure are 90° apart in phase. As a result, we can estimate the energy stored in the RF structure if we know its volume and the maximum electric or magnetic field it contains. Consider a generic RF device, shown schematically in Figure 18.

![Figure 18: Generic RF kicking structure. Beam travels along +z.](image)

The energy densities associated with electric and magnetic fields are \(\varepsilon_0 E^2\) and \(B^2/\mu_0\) when \(E\) and \(B\) are expressed in units of V/m and T.

To inject or extract a bunch, the field strength in the kicker must satisfy

\[
\langle E \rangle L \geq 3 \text{ MeV} \quad \text{or} \quad \langle B \rangle L \geq 0.01 \ \text{T} \cdot \text{m}
\]  

Recall that the average values of \(\sin(z)\) and \(\sin^2(z)\) over the interval \((0,\pi)\) are \(2/\pi\) and \(\frac{1}{2}\), respectively. If we assume the fields in the RF structure vary sinusoidally with \(z\) for a half wavelength along the structure’s length, we will have

\[
\frac{\langle E^2 \rangle}{\langle E \rangle^2} = \frac{\langle B^2 \rangle}{\langle B \rangle^2} \approx \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\pi}{4}
\]

so that
\[
\langle E^2 \rangle \sim \frac{\pi}{4} \left( \frac{3 \text{ MeV}}{L} \right)^2 \quad \text{and} \quad \langle B^2 \rangle \sim \frac{\pi}{4} \left( \frac{0.01 \text{ T} \cdot \text{m}}{L} \right)^2.
\] (9)

As a result, the energy stored in the RF device will be

\[
U = abLe_0 \langle E^2 \rangle = abL \langle B^2 \rangle / \mu_0 \sim \frac{ab\pi e_0 \cdot 9 \times 10^{12}}{4L} = \frac{ab\pi \cdot 10^{-4}}{4L\mu_0} = 62.5 \frac{ab}{L}. \] (10)

A device with \( a = b = 5 \text{ cm} \) and \( L = 10 \text{ m} \) will hold energy \( U = 0.015625 \text{ joules} \). Since the kicker is filled at 3 MHz, it requires 47 kW of RF power, neglecting losses and various coupling efficiencies. Note that the transverse electric field inside an RF structure of these dimensions is approximately 300 kV/m.

**Amplifier technology**

It appears that this power range, frequency and bandwidth suggest the use of a traveling wave tube amplifier (TWTA) rather than a klystron.\(^6\) We have not yet made more than a cursory investigation of technical issues associated with the choice of amplifier.

**Programmable function generator**

We imagine driving the RF amplifier input with a programmable function generator. The device could be reprogrammed to compensate for drifts in the behavior of the amplifier, waveguide, and RF structure.

We have not yet selected a commercial device with suitable properties.

**What is missing**

We have not modeled the coupling efficiencies between amplifier and waveguide or waveguide and RF structure. It is possible that reflections at the coupler between the waveguide and the cavity will require a circulator to be installed in the waveguide. We have not studied the effects of losses in the waveguide, or of nonlinear behavior anywhere in the system. We have not yet simulated in detail the kicker’s effect on bunches that pass through it a number of times before being extracted.

**Sensitivity to errors**

The TESLA Technical Design Report\(^7\) describes the maximum allowable kicker error as 0.07 Gauss-meters, both for bunches that are to pass through the kicker undisturbed.

\(^6\) Ralph Pasquinelli, private communication.

(when its field integral is zero) and bunches that are to be extracted. Imperfections or drifts in amplifier performance, waveguide geometry, RF structure parameters, and bunch timing will all contribute to kicker performance errors. In this section we assume that injection and extraction line correctors are installed (as discussed previously) and describe the effects of various problems on kicker error.

**RF structure Q error**

The RF structure’s response to driving fields depends on \( Q \): both the frequency dependence and the overall magnitude of the cavity fields are affected by an error in \( Q \). Shown in Figure 19 are plots of the fractional kick error as functions of deviation in \( Q \). The three curves correspond to the center, head, and tail of a bunch, assuming the “head” and “tail” positions are \( \pm 2.5 \sigma (\pm 50 \text{ ps}) \) from the bunch center along the beam’s direction of travel. Full scale deviation in \( Q \) shown in the plot is \( \pm 0.1\% \), giving rise to fractional kick errors of approximately \( \pm 0.0006 \), slightly smaller than the maximum tolerable kick error of \( \pm 0.0007 \) (0.07%).

![Figure 19. Fractional kick error caused by deviations in \( Q \) for the center (\( t = 0 \)), head (\( t = -50 \text{ ps} \)), and tail (\( t = +50 \text{ ps} \)) of an extracted bunch.](image)

The impulse error is smaller for unkicked bunches and is shown in Figure 20, in which comparisons of the effects on the kicked and first two unkicked bunches are plotted. The impulse error during an interval \( \pm 10 \text{ ns} \) around the kicked bunch is shown in Figure 21.
Figure 20. Kick error caused by deviations in $Q$ for the center, head, and tail of the kicked and first two unkicked bunches. Full vertical scale corresponds to 0.07 Gauss-meters (2.1 keV/c).

Figure 21. Kick error as a function of time caused by a $+0.1\%$ deviation in $Q$. Note that bunches are only in the kicker at integral multiples of the damping ring bunch spacing, 5.56 ns in our model.
RF structure center frequency error

The RF structure’s response will be altered if its natural frequency deviates from the nominal value. The associated error in kicker impulse is shown in Figure 22 as a function of frequency deviation for the center, head, and tail of the kicked bunch.

Figure 22. Kick error as a function of cavity center frequency error (MHz).

Figure 23. Kick error as a function of cavity center frequency error for kicked, first unkicked, and second unkicked bunches.
The effect is most significant for the head and tail of the kicked bunch; Figure 24 shows a graph of impulse error as a function of time for this bunch when the cavity center frequency is mistuned by 180 kHz (0.01%). Full horizontal scale in the plot is ±50 ps (±2.5σ).

Cavity mistuning should be maintained below 0.01% during operation.

*Waveguide length error*

Changes in the length of the waveguide will shift the arrival time of the kicking pulse at the cavity relative to the arrival time of a bunch, as well as shifting the relative phases of the different frequency components contributing to the kick. Both of these effects change the arrival time of the kick with the same sign.

Figure 25 shows the effect of a 1 mm waveguide length error. The two curves represent the ideal and delivered impulses as functions of time; full scale in the plot is ±10 ps. Note the ~5 ps shift in the positions of the peaks. Since the waveguide’s group velocity is ~2/3 c in the kicker’s frequency band, we see that most of the shift in the location of the peak is caused by the overall increase in propagation distance, rather than the dispersion-induced shift in the relative phases of the Fourier components. This isn’t surprising: the phase difference between adjacent Fourier components only changes by a few degrees per meter of waveguide: a one millimeter length error will only shift their relative phases by a few thousandths of a degree. Most of the change in the time structure of the kicking peak comes about through its delayed arrival at the RF cavity.
Figure 25. Effect of a 1 mm waveguide length error. The two curves represent the ideal and delivered impulses as functions of time; full scale in the plot is ±10 ps.

When the waveguide error is too large, the shape of the kicking peak will change so that the injection/extraction line correctors can no longer flatten the peak with sufficient accuracy. As a measure of this change in shape, we plot the difference between the delivered kick and an ideal impulse in Figure 26, in which the peaks of the delivered and ideal impulse curves have been aligned.

Most of the deviation from the ideal impulse evident in Figure 26 is due to the difference in size of the delivered kick. Rescaling the kicks to have the same magnitude as the ideal impulse yields the curves plotted in Figure 27. This is encouraging: it would be sensible to build the waveguide to an accuracy in length of a centimeter or better, but it is easy to compensate for errors (or thermally induced changes) in length by tuning the amplitude and timing of the amplifier signal injected into the waveguide.

The effect on unkicked bunches is also small, as can be seen in Figure 28. The effect on later unkicked bunches is even less than on the first unkicked bunch: as long as the waveguide length is controlled to an accuracy of a centimeter, adjusting the overall size and timing of the amplifier signal will compensate adequately for small changes in waveguide length.
Figure 26. Effect of waveguide length error. Plotted are the differences between the delivered kicks and an ideal impulse for waveguides that are 5 mm, 10 mm, 15 mm, 20 mm, and 25 mm too long. The peaks in the delivered kicks have been shifted in time to align with the peak in the ideal impulse centered at \( t = 0 \).

Figure 27. Differences between delivered kicks and an ideal impulse for waveguides that are 5 mm, 10 mm, 15 mm, 20 mm, and 25 mm too long. The peaks in the kicks have been shifted in time to align with the peak in the ideal impulse that is centered at \( t = 0 \). In addition, the delivered kicks have been rescaled to have the same magnitude as the ideal impulse.
Figure 28. Differences between delivered kicks and an ideal impulse at the arrival time of the first unkickled bunch for waveguides that are 5 mm, 10 mm, 15 mm, 20 mm, and 25 mm too long. The peaks in the kicks have been shifted in time to align with the peak in the ideal impulse that is centered at $t = 0$. The time region in the plot is centered on the arrival time of the first unkickled bunch.

**Waveguide cutoff frequency error**

Recall that the cutoff frequency for the rectangular waveguide shown in Figure 13 is $f_{\text{cutoff}} = c/(2b)$, where $b$ is the larger of the waveguide’s transverse dimensions. Inaccuracies and thermal drifts in waveguide geometry will change the cutoff frequency, and therefore the frequency dependence of the group velocity. It should be possible to correct for this by reprogramming the function generator that drives the amplifier.

The problems arising from drifts in cutoff frequency will be seen in a number of ways. The average group velocity will change, so the propagation time between amplifier and cavity will shift. This causes mistiming between the kicking peak and the beam. In addition, the phase differences between the various Fourier components arriving at the cavity are changed, leading to an error in pulse shape.

How large an error in $f_{\text{cutoff}}$ might be tolerable? We can use the sensitivity to waveguide length errors as a rough indicator. Figures 25 - 28 suggest that a centimeter length change does not distort the pulse shape by an unacceptable amount. The relative phases of adjacent Fourier components change by a few hundredths of a degree per centimeter of waveguide, so we can ask what change in cutoff frequency might have a similar effect on the phases. After traveling the full length of the waveguide, frequency components separated by 3 MHz are split in phase by typically 300 degrees. As a result, a fractional error in error in group velocity, and therefore in cutoff frequency, that is small compared
to a part in $10^4$ ought to be tolerable. Since $\delta v_g/v_g \sim \delta b/b$, we will need the waveguide transverse dimension to be stable a few microns.

To some extent we should see the same kinds of effects arising due to errors in waveguide cutoff frequency as we do with errors in length: there will be an overall time displacement of the kicking peak and a reduction in its amplitude, as well as distortion in the shapes of the impulse function in the vicinity of the zeroes. Some correction through adjustment of the timing and overall amplitude of the signal coming out of the function generator can (partially) compensate for this.

The shift in arrival time of the kicking peak is shown in Figure 29 for a 100 kHz displacement in cutoff frequency away from the nominal value of 1.3 GHz. Note the ~30 ps displacement of the peak from $t = 0$.

The influence on the shape of the kicking peak is shown in Figure 30 in which we plot the difference between the delivered kick and an ideal impulse after the peaks of the delivered and ideal impulse curves have been aligned in time, and rescaled to have the same peak amplitude. The impact on the accuracy of the zero in impulse at the first unkicked bunch is shown in Figure 31. From these figures we see that it is harder to correct the impulse function at the first unkicked bunch than it is at the kicked bunch by rescaling and shifting in time: the impulse errors 50 ps away from the bunch centers are larger for the unkicked bunch. As it stands, this naïve correction scheme will be adequate for cutoff frequency errors smaller than about 20 kHz,
Figure 30. Effects of cutoff frequency errors. The curves represent the difference between delivered and ideal impulses as functions of time after aligning the time of the peaks and rescaling the peak amplitudes. Full scale in the plot is ±100 ps. Nominal $f_{\text{cutoff}}$ is 1.3 GHz. Errors in cutoff frequency for individual curves are indicated on the plot.

Figure 31. Effects of cutoff frequency errors. The curves represent the difference between delivered and ideal impulses as functions of time. The peaks in the kicks have been shifted in time to align with the peak in the ideal impulse that is centered at $t = 0$. The time region in the plot is centered on the arrival time of the first unkicked bunch.
Amplifier gain error: linear frequency dependence

Any effect that causes the delivery of an incorrect set of amplitudes and phases for the various Fourier components to the RF structure will generate impulse errors. To see how sensitive the impulse function is to amplifier gain errors we model these errors assuming a linear frequency dependence: \( \Delta G = \alpha (f - f_{RF}) \). Naturally, other forms of frequency dependence are possible! However, since we are working with a signal whose full bandwidth is only \( \pm 10\% \) of its principal frequency, a linear model seems a sensible choice for the time being. How large can \( \alpha \) become without overly distorting the impulse function?

Figures 32, 33, and 34 show the consequences for \( \alpha \) values of 0.5% per 180 MHz, 1.0% per 180 MHz, 1.5% per 180 MHz, and 2.0% per 180 MHz. In the plots, the impulse functions have been rescaled to agree in amplitude at \( t = 0 \) with the nominal kick. Full (horizontal) scale in the figures is \( \pm 100 \) ps. Since the full bandwidth in the impulse function is \( \pm 180 \) MHz, the different curves correspond to maximum deviations from ideal gain of \( \pm 0.5\% \), \( \pm 1.0\% \), \( \pm 1.5\% \), and \( \pm 2.0\% \). As before, it is the first unkicked bunch that is most susceptible to problems: amplifier gain errors (for errors that increase linearly with distance from the center frequency) should be kept smaller than \( \pm 1\% \) over the full \( \pm 180 \) MHz band.

![Figure 32. Effects of an amplifier gain error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the kicked bunch.](image_url)
Figure 33. Effects of an amplifier gain error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the first un_kicked bunch.

Figure 34. Effects of an amplifier gain error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the second un_kicked bunch.
Amplifier phase error: linear frequency dependence

To study sensitivity to amplifier phase errors we again assume a linear frequency dependence: $\Delta \phi = \beta(f-f_{RF})$.

Figures 35, 36, and 37 show the consequences for $\beta$ values of 0.2° per 180 MHz, 0.4° per 180 MHz,… 1.0° per 180 MHz. In the plots, the impulse functions have been shifted in time to align the kicking peaks and rescaled to agree in amplitude with the nominal kick. Full (horizontal) scale in the figures is ±100 ps. As before, the full bandwidth in the impulse function is ±180 MHz.

Again, it is the first unkicked bunch that is most susceptible to problems: amplifier phase errors (for errors that increase linearly with distance from the center frequency) should be kept smaller than ~1° over the full ±180 MHz band.

Figure 35. Effects of an amplifier phase error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the kicked bunch. Full (horizontal) scale is ±100 ps. The impulse functions have been shifted in time to align the kicking peaks at $t = 0$ and rescaled to agree in amplitude with the nominal kick.
Figure 36. Effects of an amplifier phase error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the first unknicked bunch.

Figure 37. Effects of an amplifier phase error that grows linearly with frequency. The curves represent the difference between delivered and ideal impulses as functions of time. The time region in the plot is centered on the arrival time of the second unknicked bunch.
A comment on errors

The errors in impulse function caused by the effects described above can be corrected through adjustment of the signal produced by the programmable function generator. Small corrections can be made through adjustment in overall amplitude and timing of the function generator signal; larger adjustments will require changing the mix of Fourier amplitudes and phases in the signal sent to the RF amplifier.

Not all problems can be remedied through clever use of a low-level RF system to recognize impulse inaccuracies: noise and amplifier nonlinearities can introduce errors that cannot be removed through reprogramming.

Sensitivity to noise

Amplifier noise

We model amplifier noise as flat in frequency spectrum from 300 MHz to 6 GHz, and random in phase. We do not simulate noise outside this frequency range. To represent the noise voltage in the time domain, we divide the frequency band into bins that are 300 kHz wide and assign a noise voltage \( v_i(t) = v_0 \cos(\omega_i t + \phi_i) \) to each bin. The subscript \( i \) labels the bin; the phase \( \phi_i \) is selected randomly with no correlation between the phases of individual frequency bins. The overall noise voltage \( v_{\text{noise}}(t) \) is the sum of \( v_i(t) \) over all bins \( i \).

As can be seen in Figure 17c, the maximum amplifier output is \( \sim 0.015 \) in units where the kicking voltage inside the RF cavity has unit strength 1. In these units, an RMS amplifier noise voltage of \( 10^{-4} \) per \( \text{GHz}^{1/2} \) produces the results shown in the next several graphs. Figure 38 illustrates a typical noise voltage, shown as a function of time, at the amplifier output. Note the presence of high frequency contributions to the noise amplitude.

Since the RF cavity does not store energy efficiently at frequencies far from 1.8 GHz, we expect that the noise voltage inside the cavity will show much less out-of-band contribution than is present in the simulated amplifier noise. This can be seen in Figure 39 where the high frequency noise present in Figure 38 is largely absent.

The noise distribution added to the kicking impulse corresponding to an amplifier noise level of \( 10^{-4} \) per \( \text{GHz}^{1/2} \) is shown in Figure 40. Note that the RMS noise voltage in the cavity is 0.00044 (in units where the ideal impulse has unit amplitude), somewhat smaller than the limit set by the total noise budget.

The full bandwidth of the amplifier input signal is 360 MHz; restricted to this band, the rms noise associated with \( 10^{-4} \) per \( \text{GHz}^{1/2} \) would be \( 6 \times 10^{-5} \) in comparison with a maximum signal of \( \sim 0.015 \), or a required signal/noise ratio of \( \sim 250 \), or 48 dB. With a wider acceptance band (the RF cavity responds to out-of-band frequencies), the required signal/noise ratio is greater.
Figure 38. Effects of amplifier noise: the graph shows noise added to the amplifier’s output signal as a function of time over the time period $\pm 10$ ns. The amplifier is assumed to generate an RMS noise voltage of $10^{-4}$ per GHz$^{1/2}$ in units where an ideal kick has unit amplitude. Noise is generated with constant amplitude and random phase in the frequency range 300 MHz to 6 GHz.

Figure 39. Effects of amplifier noise on kicking impulse: the graph shows the cavity’s response to the amplifier noise plotted in the previous figure. Note the absence of the high frequency components evident in the previous figure. RMS noise voltage in the cavity is 0.00044, in units where the ideal impulse has unit amplitude.
noise voltage added to kicking impulse for 50,000 randomly generated noise "histories"

Figure 40. Noise voltages. RMS width of the distribution is 0.00044, in units where the ideal impulse has unit amplitude.

Arbitrary function generator noise

We do not yet model noise from the arbitrary function generator, instead including it in our naïve model for amplifier noise. It is reasonable to expect that the noise from a function generator will not be independent of frequency!

Nonlinear effects

Harmonic distortion

Our modeling of harmonic distortion is primitive, and should be further developed. At the present time we assume that harmonic distortion introduces an unwanted contribution to the amplifier output that is only a function of the amplifier’s ideal (noise- and distortion-free) output signal as well as its output noise. We do not generate harmonic distortion associated with output frequency components introduced through other nonlinear effects such as intermodulation distortion. We do not model harmonic distortion arising from imperfections in the arbitrary function generator.

As was done with the simulation of amplifier noise, we model amplifier harmonic distortion as flat in frequency spectrum from 300 MHz to 6 GHz. Only the first harmonic is generated, using a fixed phase relationship between the parent signal voltage and distortion-induced harmonic. This is more clearly (and concisely) described in equations:

\[
A_i \cos(\omega_i t + \varphi_i) \rightarrow \sum_{n=1}^{\infty} A_n \cos(n\omega_i t + n\varphi_i + \delta\varphi_n)
\]

...where \(A_n = A_n(A_i, \omega_i)\) and \(\delta\varphi_n = \delta\varphi_n(A_i, \omega_i)\).
In our model,

\[
A_n(A, \omega) = \begin{cases} 
  aA & n = 2 \\
  0 & n > 2 \\
\end{cases} \quad \text{and} \quad \phi_n(A, \omega) = \begin{cases} 
  b & n = 2 \\
  0 & n > 2 \\
\end{cases}
\]

(12)

where \(a\) and \(b\) are constants.

It would be natural to include more complicated dependences on amplitude and frequency; in the future we would like to refine these studies to incorporate more detailed properties of traveling wave tube amplifiers.

Figure 41. Voltages caused by (first) harmonic distortion. The coupling from fundamental into first harmonic is modeled to be independent of frequency as described in the text.

In our simple model, voltage arising from harmonic distortion scales linearly with \(a\) and shifts in time (without changing its shape) with \(b\). Figure 41 illustrates the unwanted voltage added to an ideal signal in the case \(a = 0.5\) and \(b = 0\). In the figure contributions to the distortion signal from amplifier noise have been omitted: it seems likely that this is normally included in the quoted values of noise amplitude quoted by manufacturers of TWTA’s.

The RF cavity response as a function of driving frequency was given in Equation 2. We see that the response to a driving signal at twice the center frequency \(\omega_0\) is
Here \( A_{ch} \) is the cavity’s response to a driving signal at its center frequency. It is helpful that the cavity’s response to signals far from its center frequency is small! Since the principal signal’s full bandwidth is only \( \pm 10\% \), the distortion-induced harmonics are far from the signal band and are tolerable when \( a < 0.5 \), corresponding to -6dB.

**Intermodulation distortion**

Intermodulation (IM) distortion can introduce spurious signals at frequencies that are the sums and differences of frequencies present in the primary signal. As was true for harmonic distortion, the signal’s \( \pm 10\% \) full bandwidth and cavity’s narrow frequency response will help suppress sensitivity to “second order” IM distortion. (Second order distortion signals primarily populate bands centered at 180 MHz and 3.69 GHz.) However, distortion-induced signals near 3.69 GHz can mix with signals in the primary band (centered at 1.845 GHz) to produce unwanted third order signals in the primary signal band.

Our modeling of IM distortion is unsophisticated at the present time. We assume that voltages created through second-order IM distortion arise from the sum of the amplifier’s ideal signal, noise voltage, and harmonic distortion signal. We do not model IM distortion arising from imperfections in the arbitrary function generator. We calculate third-order harmonic distortion signals and then include them in the amplifier output presented to the simulation.

We model amplifier couplings between different frequencies to produce IM distortion as flat in frequency spectrum from 300 MHz to 6 GHz, generating distortion signals at both the sum and difference frequencies. We describe the second order IM signals this way:

\[
A_i \cos(\omega_1 t + \phi_i) + A_2 \cos(\omega_2 t + \phi_2) \rightarrow A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2) +
A_s \cos\left(\frac{\omega_1 + \omega_2}{2} t + \phi_s\right) + A_\omega \cos\left(\frac{\omega_1 - \omega_2}{2} t + \phi_\omega\right) \tag{14}
\]

In our model,
\[ A_+ (A_1, \omega_1, A_2, \omega_2, \varphi_1 - \varphi_2) = a_+ A_1 A_2 \quad \text{and} \quad \varphi_+ (A_1, \omega_1, A_2, \omega_2, \varphi_1, \varphi_2) = \left( \frac{\varphi_1 + \varphi_2}{2} \right) + b_+ \]

\[ A_- (A_1, \omega_1, A_2, \omega_2, \varphi_1 - \varphi_2) = a_- A_1 A_2 \quad \text{and} \quad \varphi_- (A_1, \omega_1, A_2, \omega_2, \varphi_1, \varphi_2) = \left( \frac{\varphi_1 + \varphi_2}{2} \right) + b_- \]

(15)

where \( a_+ \), \( a_- \), \( b_+ \), and \( b_- \) are constants. The same expressions are used to generate third-order IM distortion.

Work on modeling of intermodulation distortion is in progress.

**Beam dynamics: effects associated with multiple passes through the kicker**

Most bunches pass through the kicker several times after injection, or before extraction. There are a variety of effects that enter into a calculation of the kicker’s influence on a bunch that orbits the damping ring several times during the injection/extraction cycle. In the extraction scheme under consideration, a bunch arrives at the kicker one “click” later each time it begins its next orbit of the damping ring until finally being ejected. As a result, it passes through the kicker during zeroes that are progressively closer to the kicking peak.

In a naïve description of the damping ring, the effects on a bunch associated with multiple zero crossings would be unimportant due to the field integral’s zero first derivative. But imperfections in the kicker can introduce distortions to the impulse function that could cause coherent effects that will spoil beam emittance. Oscillations of the beam about a closed orbit, as well as the effects of synchrotron oscillations of particles inside the bunch relative to the bunch center, can be included in a beam dynamics simulation of the kicker.

**Prototyping and tests**

We have had discussions with Fermilab’s RF group and Technical Division about studies of some of the technical challenges associated with the Fourier series kicker. After our simulations are finished we will collaborate with the lab on the engineering studies necessary to see if the concept is workable. We have submitted an expression of interest\(^8\) to Fermilab to test devices in the A0 photoinjector beamline.

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\(^8\) G.D. Gollin, M.J. Haney, and J.B. Williams, An Expression of Interest Concerning Investigation of TESLA Damping Ring Kickers using the A0 Photoinjector Beam, May 24, 2004, [http://www.hep.uiuc.edu/home/g-gollin/Linear Collider/Fermilab_kicker_A0_proposal.pdf](http://www.hep.uiuc.edu/home/g-gollin/Linear Collider/Fermilab_kicker_A0_proposal.pdf).
## UIUC Budget for LCRD 2.22

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